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XXIII. *Oscillatory Currents in Coupled Circuits. By* G. W. O. Howe, *M.Sc., Whit. Sch.*

Received March 17, 1911. Read March 24, 1911.

If a condenser with a capacity of K farads be charged and then discharged through an inductance of L henries, the resulting current is oscillatory if the resistance of the circuit is less than $2\sqrt{\frac{L}{K}}$. The oscillatory current has a logarithmic decrement of $\frac{R}{2\sim L}$ per period, and if R is small compared with L, a frequency of $\sim = \frac{1}{2\pi\sqrt{KL}}$.

By taking condensers and inductances of suitable values, these oscillatory currents can be shown on the lecture screen by means of the Duddell projection oscillograph. To do this

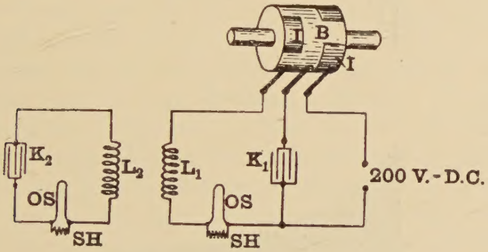


Fig. 1.

conveniently the ordinary charge and discharge key is replaced by three brushes rubbing on a special commutator which is fixed to the spindle of the oscillograph motor, so that a number of similar wave-trains are superimposed on the screen. As shown in Fig. 1, the commutator consists of a brass cylinder, B, certain parts of which are cut away and replaced by insulating material, I. During a short interval the condenser is connected across a direct-current supply, from which it is then disconnected and discharged through an inductance, in series with which is placed the strip OS of the oscillograph, suitably

shunted. This oscillatory circuit is only broken while the condenser is being charged. In the experiments shown, the capacity consists of four Western Electric Co. condensers, nominally of 5 mfds. connected in parallel, while the inductance is an air-core choking coil of 28 millihenries with a resistance of 0·32 ohm. The oscillations have a frequency of 220 per second. All the curves accompanying this Paper were obtained by placing a photographic film on the tracing desk of the oscillo-

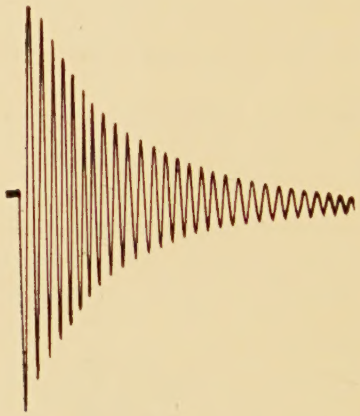


Fig. 2.

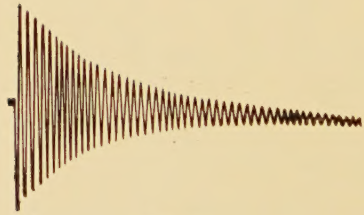


Fig. 2a.

graph and exposing for a few seconds. Figs. 2 and 2a show the curves obtained from single oscillatory circuits. The measured resistance of the circuit was in all cases about half an ohm, while the resistance calculated from the decrement is about 1·5 ohms. (This does not apply to Fig. 2a, which is referred to at the end of the Paper.) The condensers are, therefore, the main cause of damping in the circuits. By

placing a variable resistance in the circuit the damping can be increased until the circuit is made non-oscillatory.

When a circuit contains a spark-gap the conditions are somewhat modified, as the resistance in the gap, and therefore the decrement of the circuit, increases as the current decreases in amplitude.

In the absence of an oscillograph the phenomenon is most easily demonstrated by the mechanical analogy of a gravity or spring-controlled oscillating mass, with a frictional resistance proportional to the speed. It two pendulums are hung from a non-rigid support, such as a stretched cord, they are analogous to two oscillatory circuits with a certain amount of mutual inductance. In the electrical case the E.M.F. induced in the second coil is proportional to the rate of change of the primary current—that is, to what one might call the electrical acceleration. Now the acceleration of a pendulum is a maximum when it is at its maximum amplitude, and that is the moment when, by displacing the point of support, it is exerting the greatest force on the other pendulum.

We shall consider the case in which the two oscillatory circuits are adjusted to have the same natural frequency. The second strip of the double oscillograph is put in the new oscillatory circuit, and the two inductances so placed that they mutually affect each other (*see* Fig. 1). When a discharge passes through the primary circuit, energy will be transferred from the primary to the secondary circuit. If the mutual inductance between the coils is low, the rate of this transfer of energy will be small compared with the rate at which energy is frittered away by the damping in the primary circuit. The secondary current will then never be strong enough to have any marked effect on the primary circuit beyond a slight increase in its damping. In the secondary circuit we shall have a train of the natural frequency, increasing in amplitude up to a certain point and then gradually decreasing. Figs. 16 and 17, Plate I., represent such cases.

If now the coupling between the two circuits be tightened—*i.e.*, if their mutual inductance be increased—the transfer of energy takes place at a greater rate and goes on until the energy is entirely transferred to the secondary circuit, the amplitude of the primary current being reduced to zero. The action then reverses, the role of primary and secondary being interchanged until the energy has been entirely dissipated. This is shown in Figs. 3 to 17, which are all strictly comparable, as the circuits

were unchanged except by the introduction of resistance and by altering the distance between the coils. The sensibilities of the two oscillograph strips were adjusted to equality before taking the series of photographs.

Increasing the mutual inductance is represented in the mechanical analogy by decreasing the rigidity of the common support.

It can be shown mathematically that, although the two circuits may be tuned to have exactly the same periodic time, T, when oscillating alone, the oscillatory current in each circuit will now be the resultant of two superimposed harmonic oscillations of different frequencies. If T_1 and T_2 be their periodic times,

$$T_1=\sqrt{T^2+\tau^2} \text{ and } T_2=\sqrt{T^2-\tau^2},$$

where $\tau^2=4\pi^2M\sqrt{K_1K_2}$. We have already seen that

$$T^2=4\pi^2K_1L_1=4\pi^2K_2L_2.$$

The ratio $M/\sqrt{L_1L_2}$ is generally known as the coupling and designated by k, so that $\tau^2=kT^2$ and $T_1=T\sqrt{1+k}$ and $T_2=T\sqrt{1-k}$.

The superposition of these two harmonic oscillations of different frequencies will result in beats, and if T_B be the periodic time of these beats, it is evident that

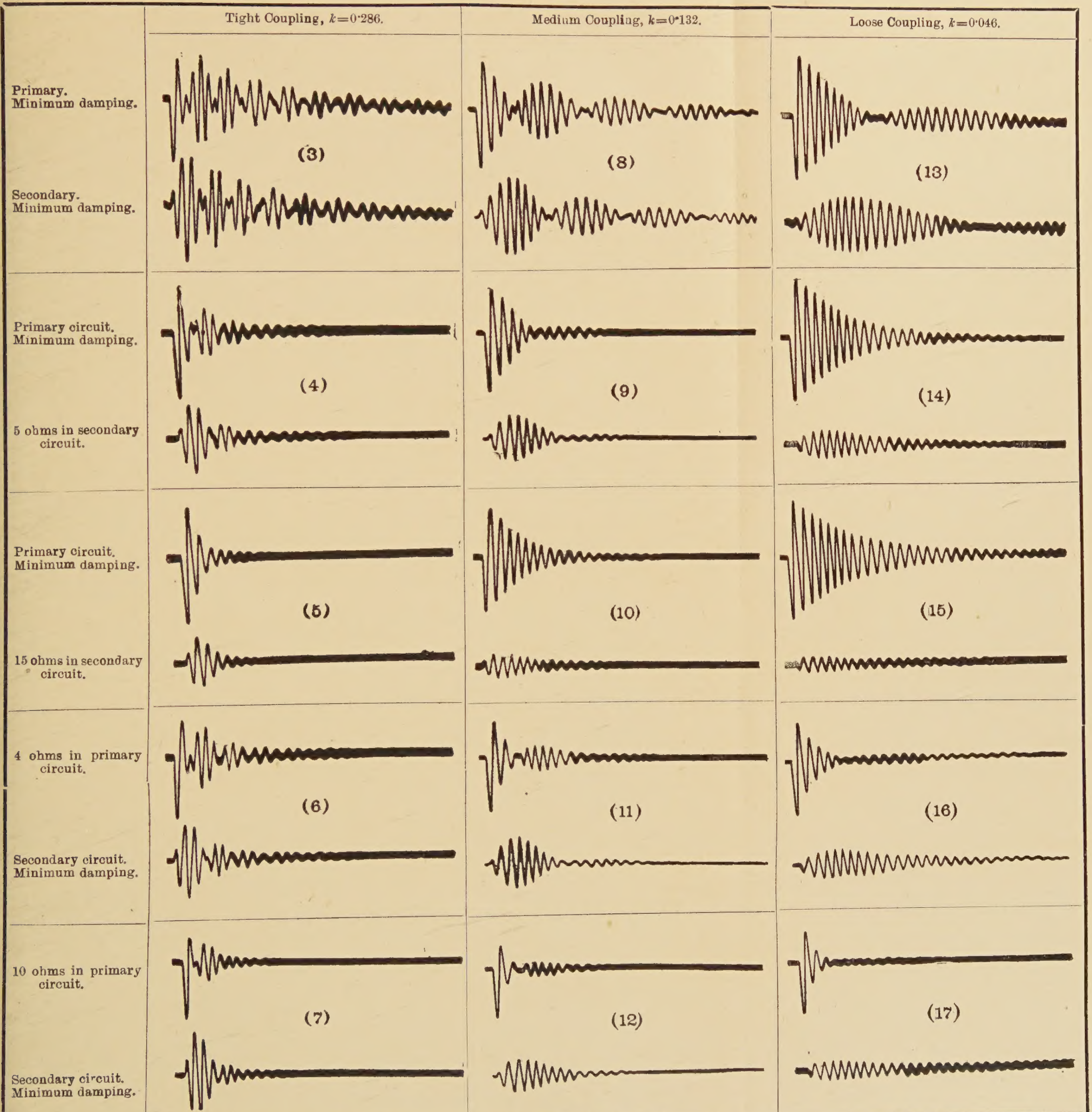
$$T_B=\frac{T_1T_2}{T_1-T_2},$$

and that the number of waves in each beat is

$$\frac{T_B}{T}=\frac{\sqrt{1+k}\ \sqrt{1-k}}{\sqrt{1+k}-\sqrt{1-k}}.$$

A half of this number will be the number of oscillations made by the primary current before the energy has been all transferred to the secondary circuit, which is very important from the point of view of the probability of the spark in a radiotelegraphic sending apparatus being quenched at this moment. In the table the number of complete oscillations per half-beat is given for various values of the coupling. It will be seen that when the coupling is less than a third, which is always the case in practice, the number of complete oscillations per beat is the reciprocal of the coupling. This will be seen to agree very well with the experimental results; for instance, in

PLATE I.



In each circuit, $K=18{\cdot}3$ microfarads. $L=28$ millihenries. R (excluding condenser losses)$=0{\cdot}45$ ohm.
R (from curves)$=1{\cdot}45$.

To face p. 240.

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$k=\frac{M}{\sqrt{L_1L_2}}$.	$\frac{T_s}{T}=\frac{\sqrt{1+k}\sqrt{1-k}}{\sqrt{1+k}-\sqrt{1-k}}$.	Number of waves per half beat.
0·05	20	10
0·1	10	5
0·2	4·88	2·4
0·3	3·15	1·6
0·4	2·24	1·1
0·5	1·67	0·84
0·6	1·26	0·63

Fig. 13 the measured coupling was 1/22, and there are about 22 waves in the beat.

Figs. 3 to 7 on Plate I. were obtained with comparatively tight coupling—viz., $k=0{\cdot}286$—the mutual inductance being 0·008 henry, while each coil has a self-inductance of 0·028 henry. In Figs. 8 to 12, k was reduced to 0·132, and in Figs. 13 to 17 it was still further reduced to 0·046. In Figs. 3, 8 and 13 the damping was small, no additional resistance being introduced. In Figs. 4, 9 and 14 a resistance of 5 ohms was in-

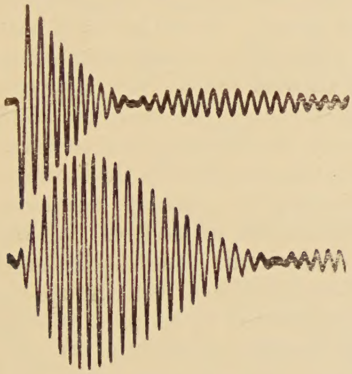


Fig. 18.

serted in the secondary, and in Figs. 5, 10 and 15 this was increased to 15 ohms. In Figs. 6, 11 and 16 the secondary circuit had the smallest possible resistance, but 4 ohms was inserted in the primary, while in Figs. 7, 12 and 17 this was increased to 10 ohms. Some of these figures could be improved by increasing the sensibility of the instrument, but this would make comparison of the different figures rather difficult. Fig. 18 shows the effect of increasing the sensibility by

partially unshunting the strips, everything else remaining the same as in Fig. 13. It will be noticed at once that the secondary maximum does not coincide with the zero point of the primary oscillation ; this is due to the fact that, after a certain point, the secondary damping dissipates energy more rapidly than it is being received from the primary circuit, so that the amplitude of the secondary current decreases in spite of the energy transfer from the primary. So far as the secondary current is concerned it is almost immaterial whether the damping resistance is placed in the primary or in the secondary circuit. The primary current is, however, very different in the two cases.

If the frequency of the oscillations is increased beyond a certain point, the Duddell oscillograph will be unable to follow the rapid variations of the current. The only form of oscillograph which can then be used is that depending on the deflection of the cathode ray in a vacuum tube. In radio-telegraphy the high-frequency oscillations are always investigated indirectly by means of a so-called wave meter, which is simply an oscillatory circuit, the natural frequency of which can be varied over a wide range. It will act as a harmonic analyser, picking out and responding to any oscillation of its own frequency in the circuit with which it is loosely coupled. Figs. 16 and 17 show exactly the conditions in a wave meter loosely coupled with a single oscillating circuit containing a spark-gap.

By placing a hot wire ammeter in the loosely coupled secondary circuit and varying the capacity, resonance curves can be plotted and the decrement calculated. As we can determine the decrement of each circuit from the curves on the screen, we can check the values calculated from the resonance curves and study the errors introduced by coupling too tightly.

By using three oscillating circuits we can represent and study the conditions when a wave meter is loosely coupled with an aerial in which there are two superimposed oscillations.

In the sending apparatus of a radio-telegraph station we are met with this difficulty. If the primary oscillating circuit containing the spark-gap be loosely coupled with the aerial, the electromagnetic waves sent out will have one definite frequency, but the transfer of energy to the aerial will take place so slowly that the major portion of the primary energy will be frittered away in the spark-gap. If, on the other hand, the circuits be coupled tightly, the energy will surge backwards and forwards between the two circuits and powerful electromagnetic

beats will be sent through the ether. This will reduce both the efficiency and the possibility of sharp tuning. The obvious way out of the difficulty is to break the primary oscillating circuit at the first moment that all the energy is transferred to the aerial, and thus makes its return to the primary circuit impossible. The possibility of doing this was discovered by Max Wien in 1906. He found that by making the spark-gap very short and thus ensuring that no part of the spark-path was very far removed from a large mass of cold metal, the disruptive strength of the gap is so quickly restored that the circuit is effectually broken if the current in it fall to a small value for a short time. The more efficient the quenching of the spark, the shorter will be the time during which the amplitude of the current must remain small in order to prevent the spark restriking, and therefore the tighter can be the coupling. This is

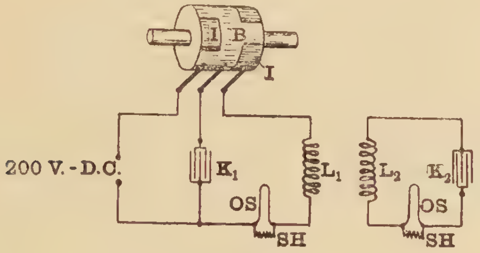


Fig. 19.

of great advantage, as it lessens the time during which energy is being dissipated in the spark.

To represent this on the oscillograph it is only necessary to break the primary oscillating circuit at the moment when the amplitude of the primary current has fallen to zero. The simplest way of doing this is to rearrange the connections to the commutator, as shown in Fig. 19.* The speed of the motor must then be adjusted so that the short interval during which the primary circuit is closed is equal to the time taken for the transfer of energy from the primary to the secondary circuit. The result obtained is shown in Fig. 20. If the quenching of the spark is not effective enough to quench it at the first opportunity it may succeed in so doing the next time the primary

* The original intention was to make a special commutator for the purpose; the possibility of simply rearranging the connections to the same commutator was pointed out by Mr. Duddell.

amplitude falls to zero, as shown in Fig. 21. In Fig. 22 the quenching occurs at the third attempt. As a matter of fact, a spark-gap would cause the decrement to be much greater than

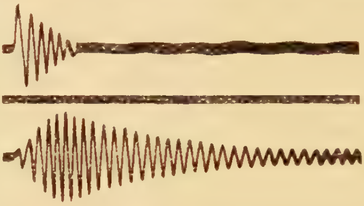


Fig. 20.

that shown in Fig. 22. The spark may be looked upon as a minimum current cut-out which we try to make as quickly-acting as possible.

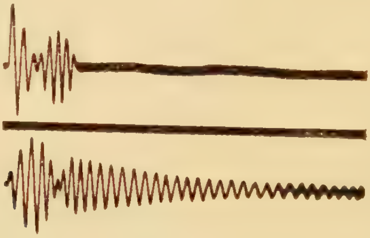


Fig. 21.

All the curves, with the exception of Fig. 2A, were obtained with Western Electric Co. condensers. The decrement in Fig. 2 gives a calculated resistance for the circuit of 1·57 ohms,

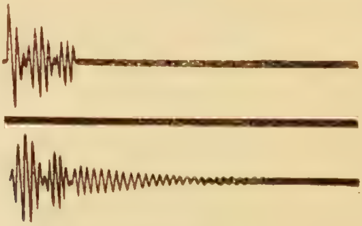


Fig. 22.

whereas the measured resistance as determined by continuous current was only 0·57 ohm. The condensers have thus an equivalent resistance of 1 ohm, and constitute by far the most

important source of damping. To see how far the damping could be reduced, these condensers were replaced by far bulkier Varley condensers of 20 mfds. Fig. 2A shows the result obtained. The speed of the oscillograph motor was reduced in order to show more of the wave-train. The decrement is much less and the equivalent resistance of the condensers is only 0·35 ohm. To show the accuracy of this method of finding the losses in condensers the following tables are given showing the determination of the damping from Figs. 2 and 2A.

The writer's thanks are due to his assistant, Mr. F. E. Meade, for his hearty co-operation in taking the photographs which accompany this Paper.

FIG. 2. Western Elec. Co. Condensers. K=18·3 mfd. p=220.					FIG. 2A. Varley Condensers. K=20 mfd. p=212.				
No.	θ	No.	θ	Ratio.	No.	θ	No.	θ	Ratio.
1	79	9	28·5	2·77	1	41·5	13	16·5	2·52
2	69	10	25	2·76	2	38·5	14	15	2·50
3	61	11	22	2·77	3	35·5	15	14	2·54
4	53	12	19·5	2·72	4	33	16	13	2·54
5	47·5	13	17	2·79	5	30·5	17	12	2·54
6	41·5	14	15	2·76	6	28·5	18	11	2·59
7	37	15	13	2·84	7	26·5	19	10·5	2·52
8	32·5	16	12	2·71	8	24	20	9·5	2·53
9	28·5	17	10	2·85	9	22·5	21	9	2·50
10	25	18	9	2·78	10	20·5	22	8	2·56
11	22	19	8	2·75	11	19	23	7·5	2·53
12	19·5	20	7	2·79	12	18	24	7	2·57
Mean value of $\theta_k/\theta_{k+8}=2·774$ Log. decrement per period = 0·127. R=2pLλ=1·57. R (excluding condensers)=0·57. Equivalent resistance of condensers=1 ohm.					Mean value of $\theta_k/\theta_{k+12}=2·537$ Log. decrement per period=0·0775 R=2pLλ=0·92. R (excluding condensers)=0·57. Equivalent resistance of condensers=0·35 ohm.				

Addendum.—Attention should perhaps be drawn to the ironless choking coils used in the oscillatory circuits. They were designed by Prof. Mather for general laboratory use. They are wound with six-cored cable, the six cores being brought out to a plug-board. The framework is of wood and fibre and entirely free from metal. The range of inductance is from 7 to 250 millihenries. The power factor at 50 cycles per second is only 0·03.

XXIV. *High-tension Electrostatic Wattmeters.* By ERNEST WILSON.

RECEIVED MARCH 1, 1911. READ APRIL 28, 1911.

IT is well known that the quadrant electrometer can be used as a wattmeter on alternating-current circuits, and various methods have been devised for connecting it to the mains. The usual method is to impress upon the quadrants an E.M.F. proportional to the current in the main circuit, and to impress upon the needle either the voltage of the mains or a voltage proportional thereto. This is accomplished by passing the main's current through a non-inductive shunt and connecting the quadrants thereto and using a potential transformer when the voltage of supply has to be subdivided. The drawback to this method is that usually about 1 volt is required across the shunt, and when very large currents have to be dealt with the energy dissipated becomes large. As an alternative, "series" or "current" transformers can be employed, in which case the drop between the terminals of the primary coil can be very much reduced; but when considerable accuracy is required, especially on low power factors, this method is not reliable.

The use of a quadrature transformer whose primary winding is in the main circuit has been successfully developed by Dr. W. E. Sumpner in connection with iron-cored instruments.* It has also been developed in connection with electrostatic wattmeters,† and a diagram of one set of the connections is given in Fig. 1. In this diagram Q is the electrometer, whose moving system is connected to the ends of a non-inductive resistance, R, which is in series with a condenser, K, across the supply mains. The quadrants of the electrometer are connected to the terminals of the secondary circuit of a quadrature transformer, QT.

The action of this instrument depends upon the equation

$$\frac{(2\pi f)^2}{\mathrm{T}}\int_0^{\mathrm{T}} \mathrm{VA}\,dt = \frac{1}{\mathrm{T}}\int_0^{\mathrm{T}} \left(\frac{d\mathrm{V}}{dt}\cdot\frac{d\mathrm{A}}{dt}\right)dt, \quad . \quad . \quad . \quad . \quad (1)$$

in which V is the voltage of the supply mains, A the amperes

* "Journal," Institution Electrical Engineers, 1908, Part 191, Vol. XLI. p. 227.

† British Patent Specifications 26,512, 1905, and 2,707, 1904.

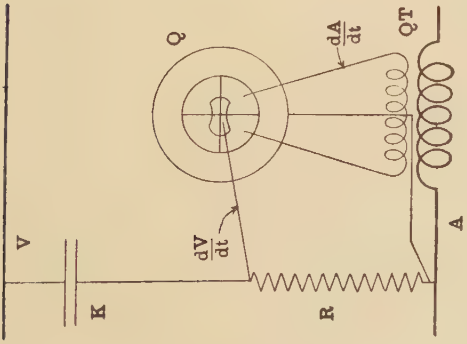


FIG. 1.

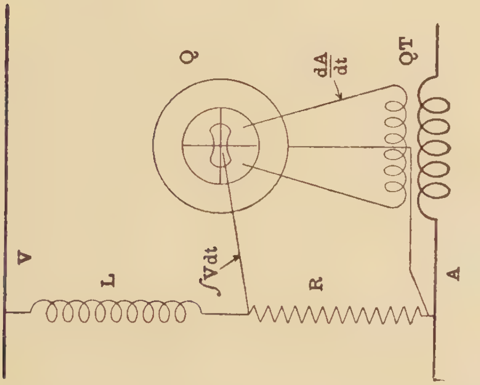


FIG. 2.

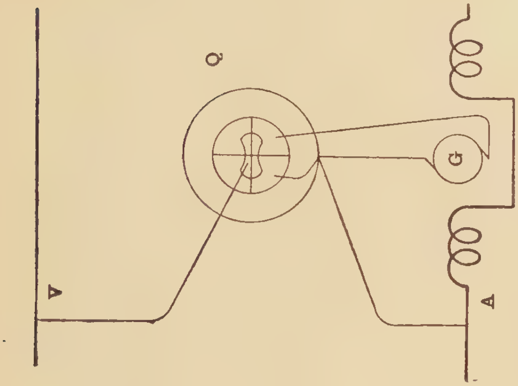


FIG. 3.

in the work circuit, f the frequency of supply and T the periodic time.

In Fig. 2 the condenser is replaced by an inductive resistance L, and in this case the action depends upon the equation

$$\frac{1}{\mathrm{T}}\int_0^{\mathrm{T}}\mathrm{VA}dt=\frac{1}{\mathrm{T}}\int_0^{\mathrm{T}}\left(\int_0^t \mathrm{V}dt\right)\frac{d\mathrm{A}}{dt}dt, \quad . \quad . \quad . \quad . \quad (2)$$

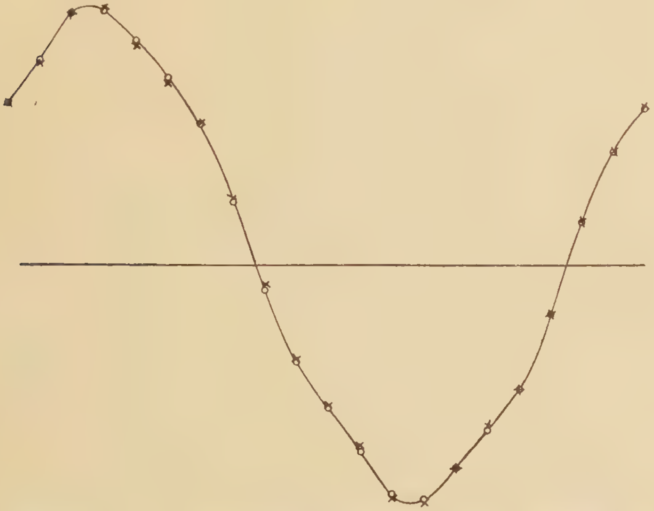
in which the differential $\frac{d\mathrm{V}}{dt}$ is replaced by the integral $\int_0^t \mathrm{V}dt$.

The arrangement shown in Fig. 1 has the advantage that the condenser K can be designed to withstand very high voltages, but it is only strictly accurate on sine curves, and equation (1) shows that its action involves the frequency f squared. A number of experiments were made with varying wave-forms, and they show that it is fairly reliable with approximate sine curves. The arrangement shown in Fig. 2 has the disadvantage that it involves an inductive resistance, L, capable of withstanding very high voltages. Equation (2) shows that it is independent of variation in wave-form. Each of these types is, of course, afflicted with the error arising from the proportion of the total voltage V to the voltage across the non-inductive resistance R, which on very high voltage systems becomes very small.

Fig. 3 shows in diagrammatic form a high-tension wattmeter whose moving system is directly connected to, and experiences the full voltage of, supply.* The quadrants are connected to a small air-cored generator, G, which rotates in a magnetic field produced by and proportional to the current A in the work circuit. This generator is driven at known speed by a small motor, which can be either direct or alternating current. It was found that when the voltage V impressed upon the moving system is of the order 20,000 volts or more, the quadrants require a considerable voltage in order that accuracy and stability may be secured. In addition, the brushes of the generator have to be carefully adjusted so as to eliminate transformer action. The following are a few particulars of one form of generator which has yielded good results: The armature is of the two-pole type, and is wound with 100 sections, each having 50 turns and an area for magnetic flux of 130 square cms. The commutator has 100 parts. The field coil is built up of copper strip 2 in. deep by $\frac{1}{32}$ in. thick, and has

* British Patent Specifications 5,582, 1904, and 27,201, 1904.

four sections connected to blocks in such wise that they can be used in parallel or series or parallel-series groupings. The number of turns per section is 10. The speed of rotation is 3,600 revs. per min., and is registered by a Hartmann & Braun speed counter, capable of detecting a variation in speed of 0·3 per cent. The curve of voltage between the generator brushes was determined for various positions of a rotating contact-maker fixed to the shaft of the alternator and compared with the curve of current in the field coil. If the generator is to be reliable, these curves should be identical. The curve shown in Fig. 4 and the observation points thereon can



O Denotes Volts. X Denotes Amperes. Frequency, 4. Speed of Generator, 3,000.

FIG. 4.

be taken as an example of the agreement obtained. The agreement is as close as can be demonstrated by the method of test.

The generator verifies the theory underlying its action. In the case before us the armature rotates in a short solenoid having a length of $4\frac{3}{4}$ in.—*i.e.*, there is a distance between the upper and lower sections of $\frac{3}{4}$ in. Under these conditions the armature which has a diameter of 4·5 in. rotates in a field which has not uniform strength, but is denser at the centre of the solenoid than near its ends. A field coil has been constructed having its conductors wound on the surface of a cylinder concentric with the armature, and fulfilling the condition that

the ampere-turns per unit length of an axis normal to the plane of the windings are constant. The field inside this winding is constant in strength and has the value $\frac{8\pi^2 a^2 n x}{3}$,* where the total turns are $2na$, n being the turns per unit length of the axis, and x the current. This theoretically perfect method of winding is difficult to accomplish when currents of 200 amperes have to be carried, as was the case in the generator first alluded to. It would appear that strip copper $\frac{1}{32}$ in. thick gives at ordinary frequencies an unobservable error due to eddy currents.

As regards the electrometer, it may be mentioned that in its first form† it resembled Lord Kelvin's quadrant electrometer. The suspension was made of phosphor-bronze strip, and the vanes, two in number, were each $10\frac{3}{4}$ in. long by about 4 in. broad at their widest part. In a later form of the instrument‡ the quadrant pairs are the moving system, and are suspended by phosphor-bronze strip between two fixed sectors, which are heavily insulated and capable of withstanding several fold the normal voltage of the instrument. The suspended quadrants are $9\frac{1}{2}$ in. in diameter, and are $1\frac{3}{4}$ in. from the fixed sectors, the instrument being designed for 15,000 volts. The instrument obeys the straight-line law, is accurate on low power-factor, and has a sensibility such that, on a scale 50 in. distant from the mirror, the kilowatts per millimetre deflection are 1·82 when the generator field coils are four series.

A further piece of apparatus, depending upon the charging of condensers in parallel and the placing of them in series, has been used for multiplying a small voltage produced by the passage of the main's current through a low-resistance shunt.§

ABSTRACT.

When using the Electrometer as a wattmeter it is necessary (in order to secure accuracy) that the voltage impressed upon the quadrants shall not be less than a certain minimum depending upon the voltage to be impressed upon the moving system. When the latter voltage is of the order 10,000, the quadrants require a voltage larger than can economically be provided by a shunt. One is led, therefore, to consider intensifying devices.

The "series" or "current" transformer, whose secondary winding is closed on a non-inductive resistance, can be used to give

* Maxwell's "Electricity and Magnetism," Vol. II., §675, Edition 1873.

† British Patent Specification 2,707, 1904.

‡ British Patent Specification 27,201, 1904.

§ *See* British Patent Specification 2,850 of 1905.

fairly good results, but it is not accurate at all frequencies and is dependent upon wave form.

The author's quadrature transformer is a very simple piece of apparatus which can be relied upon to give for electrostatic wattmeters an electromotive force which is strictly the differential of the current in the primary winding. When so used it is necessary, for accuracy, at all frequencies and on all wave forms, that the integral of the mains voltage shall be impressed upon the moving system, although for sine curves only the differential need be impressed instead of the integral.

The best device to impress on the quadrants a suitable voltage in phase with the currents is a generator with an air-cored magnetic circuit as described in the Paper. The mains current is passed through the field coil of the generator and produces a magnetic field proportional to the current; the armature is driven at known speed in this field, and is provided with a commutator and brushes. The brushes when set accurately have a voltage between them proportional to, and having the same wave form as, the mains current.

Another device depending upon the charging of condensers in parallel and the placing of them in series has also been used for multiplying a small voltage produced by the passage of the mains current through a low-resistance shunt.

DISCUSSION.

Dr. Sumpner pointed out that the reason why a fairly high voltage was needed on the quadrants, together with a strong control on the needle when a very high voltage was applied to the latter, was because of the action between the needle and the case; unless the case was quite symmetrical about the axis of the needle.

Dr. Russell inquired whether the author was troubled about brush discharges at the edges of the needle.

Mr. E. H. Rayner remarked that he had been using a quadrant electrometer for measuring the energy losses in insulating materials with a sensitiveness 1,000,000 times as great as Prof. Wilson's, giving 0·5 mm. scale deflection per microwatt. The use of a high inductance would introduce errors with low power factors, owing to its resistance. The method of working the needle at half the main voltage, having the effect of eliminating the correction for the current resistance, has such advantages that it is in many cases the natural one to use. If a dividing resistance is employed, it will be found that when high voltages and resistances are used the capacity and current of the needle will be comparable with that in the resistance, and at low power factors the error may be serious. The best way to avoid it is to connect the needle to the middle point of the high-voltage transformer, when such is used; the resistance of its winding is much less than that of any practicable dividing resistance. If a high-voltage transformer is not used for obtaining the potential, a potential divider may be made by connecting two similar switchboard voltage transformers in series across the mains and connecting the needle to the middle point. There is another error when using a high resistance of the order of 50,000 ohms in the "current" circuit when measuring a watt or less. This is caused in a similar manner to the above. The capacity current between quadrants and needle, in the case of one set of quadrants (assuming one pole of the supply is earthed), has to traverse the current resistance. This causes a deflection on open circuit which is proportional to the resistance used. Like the previous error, it may become quite important at low power factors.

Mr. C. C. Paterson drew attention to the serious limit of accuracy imposed by not being able to keep the speed constant.

Mr. W. Duddell asked if the voltage could not be reduced by means of another quadrature transformer in series with a high resistance, instead of being applied direct to the needle.

Prof. C. H. Lees drew attention to the advisability of reducing the voltage by some means before applying it to the needle.

The Author, in reply, stated that Dr. Sumpner's remarks were very interesting, as they bore out the statement in the Paper about the minimum voltage to be impressed upon the quadrants. In the case of the instrument exhibited, the *moving* system was at *low* potential relatively to the case. This reversal of the ordinary arrangement helped matters in the above connection, and gave greater accessibility to the moving parts. Dr. Russell raised the question of brush discharge. The author found that brush discharge gave rise to unsteadiness. It could be discovered by observing the instrument in the dark with the high voltage impressed, and where found the radius of curvature could be then increased and thus remove the discharge. The author was glad to hear from Mr. Rayner that he had an electrostatic wattmeter of such great sensibility. The suspension used in the case of the instrument exhibited was of very stout phosphor bronze. If necessary, the sensibility could be greatly increased.

When charging the needle, by aid of a non-inductive ratio, at half the voltage of the mains the author could very well believe that when the latter was of the order of 10,000 the capacity current would be large compared with the current in the reducing ratio due to its resistance. This would then give rise to inaccuracy. In Figs. 1 and 2 the voltage on the needle was about 100 when the voltage of the mains was 10,000. It was no doubt a very difficult thing to build an inductive resistance, L, with an air core whose CR component was small enough to render correction negligible. Replying to Mr. Paterson, he wished to say that the speed need not be regulated to any exact figure. All that was necessary was to be able to regulate it so that it came within the range of the speed meter, and then to apply a correction if necessary. Mr. Duddell suggested *two* quadrature transformers—one for the needle in series with a high non-inductive resistance, and one for the quadrants. This would only give accuracy on sine curves. Non-inductive resistances for 10,000 volts or more were very expensive, and the author had avoided their necessity. Potential transformers as a means for reducing voltage were not strictly reliable when great accuracy was desired.

XXV. *Previous Magnetic History as Affected by Temperature.* *By* PROF. E. WILSON *and* L. C. BUDD.

RECEIVED MARCH 8, 1911.

IT is well known that if a piece of iron be subjected to a considerable magnetising (previous history) force, and then be tested for permeability corresponding to a lower force, the permeability so obtained may differ widely from the permeability which would have been obtained had the material been previously demagnetised. The effects of previous magnetic history upon permeability and the dissipation of energy by hysteresis have already been studied* at ordinary atmospheric temperature. The object of the present Paper is to examine the effect of variation of temperature upon the influence of large previous history. The material used has been in the form of rings built up from stampings of Stalloy, which is an alloy of iron containing about 3 per cent. of silicon. The stampings are about 0·42 mm. thick, and have internal and external diameters of 32 mms. and 45 mms. Ring 1 was used for high temperatures, and had its windings insulated by hand with sheet asbestos. Ring 2 was used for low temperature, and had its windings insulated with silk. The following are particulars of the rings :—

—	Area of core in sq. cms.	Primary turns.	Secondary winding. Turns.	Secondary winding. Resistance in ohms at 7°C.
Ring 1	1·88	30	42	0·26
Ring 2	4·16	47	72	1·66

The method of test employs a ballistic galvanometer, and the temperatures are based upon a measurement of resistance of the secondary windings, care being taken that the magnetising force thus employed did not exceed the smallest reversal force used in the experiment. At atmospheric temperature and the temperature of liquid air the procedure was to determine the ordinary **BH** curve after the material had been carefully demagnetised. This was done by gradually reducing the primary current given by an alternator at 50

* *See* "Journal," Institution of Electrical Engineers, Vol. XXXIV., Part 170, p. 55, and Roy. Soc. "Proc.," A., Vol. LXXXIII., p. 1.

frequency, which operation took several minutes to perform. When dealing with high temperatures the ring was placed in a furnace heated by gas, and as the temperature was not absolutely steady in any one experiment it was necessary to apply the previous history force immediately after taking the observation on the demagnetised ring, and then to demagnetise again for the next point, and so on. In all the experi-

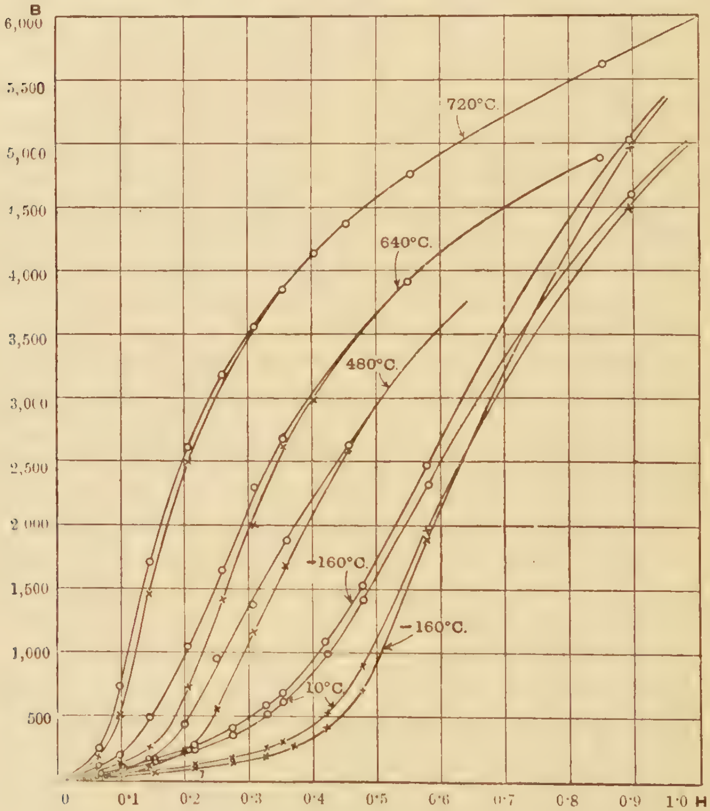


Fig. 1.

ments a previous history force of about 20 C.G.S. units has been employed.

Fig. 1 gives a set of curves taken at various temperatures with and without previous history. In every case the upper curve was given by the demagnetised material, and the difference between it and the lower one is due to previous

history. It will be seen that in liquid air the effect is most marked, and that it becomes smaller as the temperature increases. The material becomes non-magnetic at about 850°C., and regains its magnetic quality on cooling at 830°C. It seems safe to conclude that just before the temperature at which it becomes non-magnetic is reached the previous history effect has either vanished or become very small indeed.

In Fig. 2 the ratio of the magnetic inductions $\mathbf{B} : \mathbf{B}_1$ for a given value of the reversal force $\mathbf{H}$, with and without previous

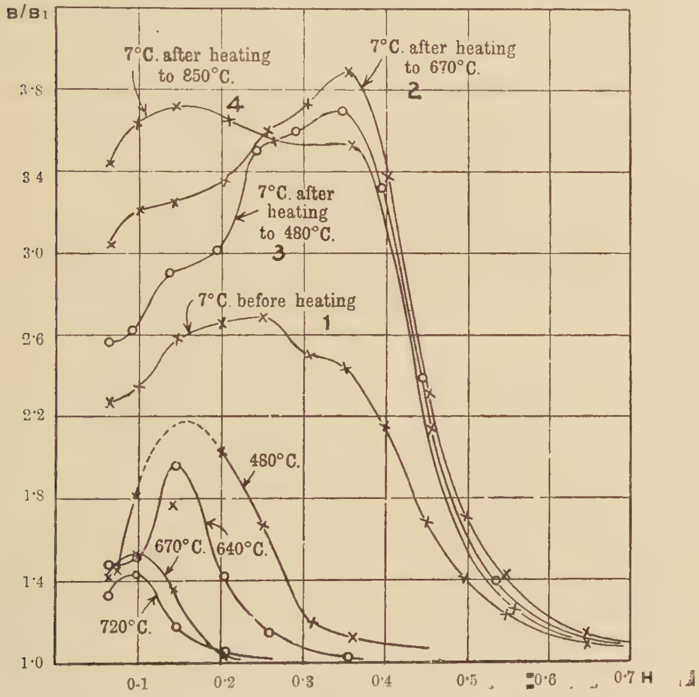


FIG. 2.

history, is given. As received from the makers the material of ring 1 gives, at atmospheric temperature, curve 1. Atmospheric temperature curve 2 was obtained after heating to 670°C. and allowing the material gradually to cool. It will be seen that previous history effect is considerably greater after the specimen has been heated, and the other curves also confirm this, the increase being due primarily to a diminution of the permeability after application of previous history. With

regard to the cooling experiment and ring 2, the curves in Fig. 3 show that after the material has been cooled in liquid air, the previous history effect at atmospheric temperature is greater than when first received, the increase being due to increased permeability when in the demagnetised condition. In spite of this, the effect of previous history is greater at the temperature of liquid air than at atmospheric temperature, and it is less at high temperatures.

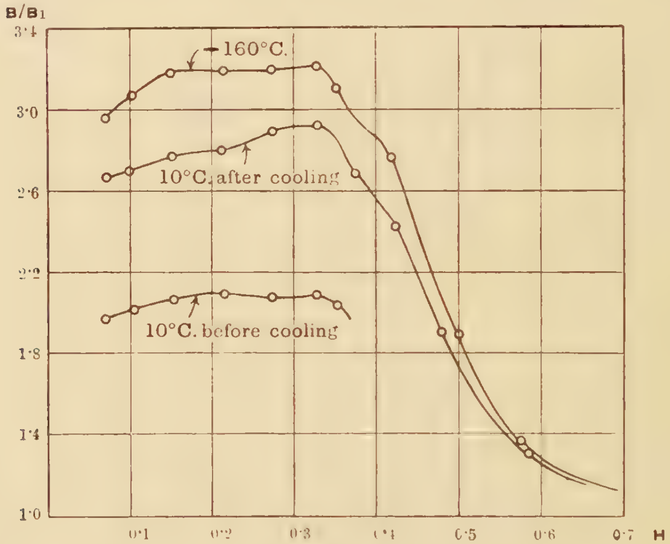


Fig. 3.

The conclusion is that the influence of large previous magnetic history in a 3 per cent. silicon alloy of iron is seriously affected by variation of temperature, being largest at the low temperatures, and ultimately becoming very small, it not vanishing, at the point near which the material becomes non-magnetic. In addition, both heating and cooling have permanent effects upon the magnetic quality of this alloy. Previous heat treatment has also to be considered.

XXVI. *Notes on the Behaviour of Incandescent Lime Kathodes.* *By* R. S. Willows, *M.A., D.Sc., and* T. Picton, *M.A., B.Sc.*

Received March 13, 1911.

Since Wehnelt's discovery that oxides of the alkaline earths emit a large number of negative ions when heated, these substances have been largely used as the kathodes of vacuum tubes. They confer the great advantage that a discharge may be passed by a P.D. as low as 30 volts. Such tubes have been shown before the Physical Society by Mr. Campbell Swinton. In one form or other they have been suggested as oscillation valves for the rectification of small oscillatory currents. As their erratic behaviour at starting frequently results in serious inconvenience, and as they are said to lose their efficiency with continued use, it was thought that a more detailed study of these points might be of interest. It was hoped, also, to throw light on the origin of the negative ions. With respect to the latter point, since the strongly electro-positive metals are found to emit negative ions freely under various conditions, the most obvious source is the metal of the oxide used. This view has been negatived by Horton's experiments,* which show that the current from calcium metal is much less than from the oxide.

The substance used, generally lime, was placed on a metal strip, platinum or nickel, either by heating the nitrate or smearing on a paste of the oxide in water. The strip was heated electrically, in a tube such as has been later used by Garrett† for experiments on aluminium phosphate; it was joined to the negative pole of a suitable battery, which drove the negative ions on to a metal plate connected with a galvanometer, the other terminal of which was joined to the positive pole of the battery. The source of the lime appeared to be without influence on the results. The temperature, when necessary, was obtained by means of a thermo-junction fused to the middle of the metal strip.

The experiments have extended over three years; they have been conducted at pressures of (1) 0·002 mm.; (2) pressures greater than 0·5 mm.; (3) in flames. The results are different according as the voltage used is greater or less than the saturation voltage when such occurs.

* Horton, "Phil. Trans.," Sec. A, 207, p. 149, 1907.
† Garrett, "Phil. Mag.," October, 1910.

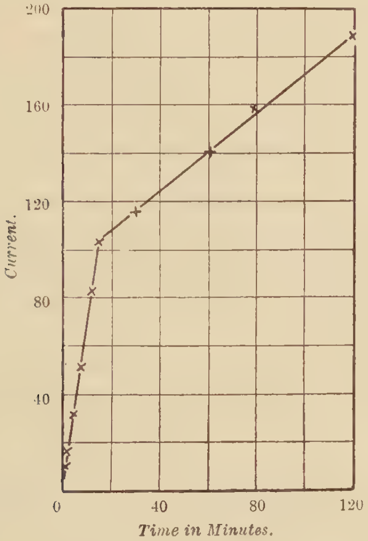
Experiments at Reduced Pressure.

Irregularities in Starting the Discharge.—It is often difficult to get much current to pass when the apparatus is first set up; this is especially the case with platinum foil as the support, when using nickel the difficulty was not often met with. A much higher temperature or voltage may be used to start the discharge, after which much lower values of either will give as great a current as is necessary. This procedure often results, unfortunately, in the fusion of the strip. A much safer way is to pass the discharge from an induction coil for a few seconds. The current may be increased by more than a hundred-fold by this means.

It is usual to regard the emission of the ions as being purely a temperature effect. In numerous instances other causes than temperature greatly influence the current. A typical instance may be given. A tube which had been started as above carried current for several hours during the first day. The following morning, when the same conditions as to temperature, pressure and voltage were established, no current passed for several minutes, when suddenly the galvanometer deflection went up to 100 divisions and oscillated violently, finally coming back to zero for some minutes. This behaviour was repeated several times during the first 20 minutes, after which the deflection gradually rose from zero and became steady in the neighbourhood of 100 divisions. During this stage, and frequently at later stages, the current is very sensitive to mechanical disturbance. A slight vibration suffices to reduce the current to zero, or to send it from zero to its full value. Similar behaviour has been noticed with the same strip, each day the tube was used, over a period of several months. We have been unable to determine the conditions necessary for sensitiveness to vibration.

Two other observations which may have some connection with one another may be mentioned. At the higher pressures the gas gradually disappears from the tube as the current is continually passed. At these pressures, 1 mm. or greater, if the lime be first heated before the battery is put on at starting, a current passes which is some hundreds of times the steady value; this rapidly decreases. The same occurs if, after heating for some hours, the battery is cut off but the heating continued. On again applying a P.D. a greatly increased current is observed, as if, in the interval, ions had accumulated

in the line only to be driven out when the battery is put on. A curve showing the course of this accumulation is shown in the figure. After getting the current to its steady value, the battery was cut off for some minutes, the heating being continued. The battery was then reapplied and the galvanometer read as quickly as possible. The current was passed until it again became steady, when the battery was again cut off, to be reapplied at a longer interval. The abscissæ represent the times the ions were allowed to accumulate, the ordinates the initial currents. These two effects have not been



noticed at pressures as low as 0·002 mm., nor with voltages less than the saturation voltage.

Alteration of Activity with Time.

Conditions of Experiments.—Lime spread on platinum strip, pressure 0·002 mm. and upwards, voltage 36, which is greater than the saturation voltage, temperature 1,100°C.

Under these conditions, neglecting the initial irregularities, the current gradually increased for two hours, after which it remained steady for the further five or six hours it was heated. The apparatus stood without heating over night. The initial current on the second day was rather higher than on the first,

and finished also at a higher value. During the night there was always a loss of activity. This behaviour was repeated for some months, until the platinum fused. The final activity was considerably greater than that at the beginning. One record out of many may be briefly given.

February 24th.—Initial current=50; after six hours heating current=130.

February 25th.—Initial current=60; after six hours heating current=160.

March 10th.—After heating for six hours current=370.

May 18th.—Current=460.

On the last date air got into the apparatus. After this had been pumped out the current was only 32 divisions, but gradually rose again, and apparently started to run through the same cycle of changes. Admission of fresh gas, whether air, hydrogen or carbon dioxide, always produced this sudden fall in current, no matter at what voltage or pressure the observations were being conducted. The recovery was more rapid with hydrogen than with the other gases, especially if it were allowed to stand for some hours in the apparatus. So far, then, from becoming fatigued with time, the lime actually increased in activity ninefold.

Observations with a Nickel Strip.—The results with a nickel strip were rather different. On account of the frequent fusion of the nickel the experiments could not be extended over such long periods.

It was much easier to obtain a visible discharge with nickel than with platinum. A P.D. of 36 volts usually caused a strong visible discharge to pass after a few minutes heating. Under these circumstances the current rose, generally, to a maximum after two hours and then decreased. Upon restarting next day the current was usually, but not always, much greater than at the *close* of the previous day's heating; it rose to a higher maximum and then decreased as before. This behaviour was repeated daily, until the final activity was sometimes 200-fold greater than at the start.

When we were fortunate enough to keep a nickel strip on for more than a week, the gas pressure remained very steady for some days and finally decreased, as in an ordinary vacuum tube.* With platinum there was a steady evolution of gas over three months. A careful spectroscopic examination of

* Willows, "Phil. Mag.," April, 1901, p. 503.

the gas in the tube might throw light on the production of the ions; the requisite apparatus was not, however, at our disposal. To test roughly whether the gas in the tube was being changed, an auxiliary vacuum tube was sealed on, and the voltage at the terminals of this was tested with a small coil at intervals. A small rise was observed, but this might be due to the electrodes altering on account of gas escaping from the metal.

The different results with the two metals suggest some chemical interaction between metal and lime. We hope to examine this point later. It was hoped to put such change in evidence by measuring the resistance of the strip day by day. A Crompton potentiometer was used for this purpose, but no change of the order 1 in 1,000 could be found.

Experiments with P.D.s less than the Saturation Voltage.

The experiments at the higher voltages will evidently be influenced by the nature and pressure of the gas, since a large part of the current is carried by ions formed by collisions. With voltages less than that required for saturation the gas carries none of the current.

In one case, typical of the rest when nickel is used, the saturation voltage was 20; the currents were, therefore, measured with 17 and 36 volts. At the lower voltage the current gradually increased over some days to double its initial value, while the usual rise to a maximum and subsequent fall was found with 36 volts. The current due to 17 volts showed no alteration when the apparatus was allowed to stand over night; the current from 36 volts, as already mentioned, showed a large increase.

Using a platinum strip and the lower voltage, the current did not vary by more than 10 per cent. over a month; afterwards a slow increase was observed. No increase or decrease in activity occurred during the night.

Experiments in Flames.

A lime electrode in a flame has been suggested in France as a detector for waves; a few observations on its fatigue may be of interest.

Two platinum electrodes, one of which was coated with lime, were placed in the flame of a Meker burner, and, by means of a

suitable battery, the negative ions were driven on to the blank electrode, which was connected to a galvanometer. The current so obtained fell in 10 minutes to one-tenth of its initial value, and then remained more or less steady for the day. The lime had completely recovered its activity next day and went through the same sequence of changes. This large initial rush of current each day is exactly similar to that already found in vacuum tubes at the higher pressures. It was thought it might be due to the oxide absorbing carbon dioxide to form the carbonate: the electrode was therefore kept on successive nights in air which had been freed from carbon dioxide, in hydrogen and in carbon dioxide, but no difference in behaviour was found.

Summary of Results.

1. When lime is heated on platinum foil, so far from showing fatigue, it actually increases in activity. With P.D.s greater than the saturation voltage this increase may be ninefold. At lower voltages a slow but steady increase up to 100 per cent. has been found. The steady activity falls when the lime is cold, the initial activity may, however, greatly increase.

2. With nickel foil, if the tube carries a heavy discharge, the current increases to a maximum and then decreases. A greatly increased activity is frequently shown after the lime has been cold for some hours. At the lower voltages the current shows the same general variations as when platinum is used.

3. Great irregularity is frequently shown on starting the current in a tube that has not been used for some hours. The li e may be at 1,000° C. and yet produce no appreciable current. At this stage the discharge is very sensitive to mechanical vibration. A discharge from a coil greatly increases the activity.

We desire to acknowledge the assistance rendered by Mr. F. G. Bratt during the course of the investigation.

ABSTRACT.

Wehnelt has shown that incandescent lime emits a large number of negative ions; if, therefore, hot lime is used as the kathode, a discharge may be obtained in a vacuum tube with P.D.s as low as 30 volts. The alteration with time of these kathodes, under continued use, has been investigated and the following results obtained: (1) When lime is heated on platinum foil, so far from showing fatigue, it actually increases in activity. With P.D.s greater than the satura-

tion voltage this increase may be ninefold. At lower voltages a slow but steady increase up to 100 per cent. has been found. The steady activity falls when the lime is cold, the initial activity may greatly increase. (2) When the lime is heated on nickel foil, if the tube carries a heavy discharge, the current increases to a maximum and then decreases. A greatly increased activity is frequently shown after the lime has been cold for some hours. At the lower voltages the same general variations are shown as with platinum. (3) Great irregularity is frequently shown when the current is first started; at this stage other causes than temperature, such as mechanical vibrations, greatly influence the emission of ions.

DISCUSSION.

Prof. C. H. LEES asked whether the Author had examined the film under the microscope. He wanted to know whether there was any likelihood of the film getting detached from the platinum and also whether the increase of activity was simply dependent upon the quantity of electricity passed.

Dr. WILLOWS, in reply, stated that he had examined the films with a microscope. The films were very adherent to the platinum. It was impossible to render the platinum free again from the activity with which the lime endowed it. There was no connection between the activity and the quantity of electricity passed. The increase of activity was probably due to the CaO diffusing into the platinum.

XXVII. *On the Formation of Dust Striations by an Electric Spark.*

By S. MARSH, *B.Sc., Ph.D., late Fellow of University of Wales, Lecturer in Physics at Battersea Polytechnic: and* W. H. NOTTAGE, *B.Sc., Demonstrator in Physics at Battersea Polytechnic.*

COMMUNICATED BY W. THOMSON. RECEIVED MARCH 14, 1911.

§ 1. In the "Phil. Mag." for November, 1909, Mr. T. J. Richmond describes some experiments on the formation of striations in a tube containing a thin layer of fine powder by means of an electric spark which passed between terminals placed at the end of the tube. He showed that the striæ distances varied with the diameter of the tube, and were further dependent upon the type of electric discharge and upon the nature of the powder. The striæ distances for different oscillatory circuits were measured, but the values obtained bore no obvious relation to the frequencies of the electrical oscillations.

In the present Paper the results of an investigation concerning the formation of the striations produced by a spark are given.

It seemed highly probable that the striations produced by the discharge were of the same nature as the striæ seen in Kundt's tube, and experiments were made with a view to investigating this point.

It has been shown by König* that if two spheres be present in a moving fluid at a moderate distance apart they repel one another in the line of motion of the fluid, and attract in a direction perpendicular thereto, and he showed, further, that this offers an explanation of the striations seen in Kundt's tube. The theory has been recently confirmed, and the law governing the striæ distance investigated by Dr. J. Robinson.†

In working with tubes complications arise owing to the reflections of the waves at the walls of the tubes. It was found‡ that well-defined striations are formed by a spark discharge between two terminals placed over a horizontal plate on which

* "Wied. Annalen," 42, 353, 549, 1891.

† "Phil. Mag.," July, 1909; April, 1910.

‡ We have recently discovered some records of experiments on striæ formation by an electric spark in various numbers of the "Berichte der Wienerschen Akademie," 1875-8, by Mach, Rosicky and others. They used horizontal sparks and coated the plate with lamp black. These observers attributed the formation of the (few) striations seen to acoustical causes.

a fine powder is sprinkled. The striations can be obtained on plates of different materials—*e.g.*, glass, ebonite, sheet tin, cardboard, &c. Some preliminary experiments were made with horizontal sparks at various distances above the plates and others where the spark was vertical and at different distances above the plate. Finally, a vertical spark was employed, which passed through a hole bored through the centre of the (glass) plate, the lower terminal being just below the plate.

§ 2. Investigations were made as to the consequences of the assumption that the periodicity of the sound waves producing the striations was determined by the frequency of the electrical circuit. It was found that the striæ distance was of the same order of magnitude as (and often greater than) the wave-length on this supposition, which fact gives rise to difficulties in the mathematical treatment. We shall refer to these later on. Further, the work of Richmond and a series of experiments performed by ourselves showed no relation between frequency (electrical) and striæ distances.

Now the discharge of electricity across the spark-gap in an oscillatory circuit consists of a number of groups of oscillations separated by relatively large intervals. Moreover, the pulses of the individual groups are heavily damped, and the value of the current, and consequently the heating effect, falls off very rapidly. In order to produce the striations with any degree of clearness and regularity it is necessary to employ a powerful spark, and it seems probable that only the first few pulses in each group are active. In other words, it is supposed that each group of oscillations produces an intense condensational pulse. The induction coil itself gives rise to a quasi-periodicity, in that a certain number of these groups is produced every second. But the fact that the striations are formed by a single group given by a Wimshurst machine and Leyden jar shows that this quasi-periodicity is not material to their formation.

Supposing, then, the heating in the group of oscillations to send out an intense condensational pulse, this would be followed by a rarefaction, and that in turn by a condensation, and so on, a short train of progressive waves being produced, the wave-length of which would depend upon the initial condensation. Concerning this point there are some results to hand. In the well-known experiments of Töpler on the wave sent out by an electric spark, using the so-called "Schlieren Methode," the air pulse appears to be very narrow; but

Rosicky and Mach,* using an interferometer method, have shown that the pulse is by no means so "thin" as was supposed. Rosicky concluded from his observations that the condensational pulse often had a thickness of 1 cm. or more. Further, the experiments of Altberg † on sound waves of short wave-length do not, in our opinion, speak against this view. The spark he used was very short (0·5 to 1 mm.). Again, his conclusions only hold for $\lambda > 2$ mm., which is a value considerably in excess of that for a Leyden jar discharged through a short wire. Further, in determining the length of the sound waves he used a diffraction grating. The grating will pick out a period in the initial disturbance falling on it, and seeing that each group of discharges has a periodic constitution, the grating transmits the disturbance with this periodicity emphasised. For circuits of very small frequencies the sparks in each group of discharges follow at such relatively long intervals that they can no longer be regarded as culminating together to send out an intense thick pulse. Consequently, one would expect to hear a note whose frequency was determined by that of the electrical circuit.

§ 3. We suppose, therefore, that a short train of progressive waves is sent out by the discharge. The length of the spark-gap varied between 1·5 cm. and 2 cm. No advantage is gained by making it longer, for the spark then takes a very irregular path, and, further, is much too "thin" to produce good striæ. Further, we assume that the waves sent are spherical in type (*see* later), and proceed to develop an expression for the interval between consecutive striæ λ and their distances from the spark on that assumption.

§ 4. König has shown (*l.c.*) that the force of repulsion between two spheres of radii a_1 and a_2, which are at rest in a current of velocity w, is equal to

$$-3\pi\rho a_1^3 a_2^3 \cos \quad (3-5\cos^2\theta)\, w^2/r^4,$$

θ being the angle between the line of centres of the spheres and r their distance apart. ρ is the density of the fluid. Where the current is alternating we have to take the mean values of w^2. If w_1 and w_2 are the respective velocities of the current at the spheres, then the mean value of $w_1 w_2$ must be taken.

It should be noted that where the velocities differ in phase by more than a quarter of a period there is a force of *attraction*

* "Berichte der Wien. Akademie," 1878, 78.

† Altberg, "Annalen der Physik," 23, 1907.

between the spheres. Where the phase difference is less than a quarter period the spheres repel.

Adopting the notation of § 263 in Lamb's "Hydrodynamics" (2nd edition), the differential equation for the propagation of spherical waves is

$$\frac{\partial^2 r\varphi}{\partial t^2}=c^2 \cdot \frac{\partial^2(r\varphi)}{\partial r^2},$$

φ being the velocity potential at a distance r from the centre.

A solution of the equation is

$$\varphi=\mathrm{A}\frac{\cos(\sigma t-\kappa r)}{r},$$

where the velocity of propagation $c=\frac{\sigma}{\kappa}$, the wave-length $\lambda=\frac{2\pi}{\kappa}$, and period $\mathrm{T}=\frac{2\pi}{\sigma}$.

The particle velocity $=-\frac{\partial\varphi}{\partial r}$, and we have

$$-\frac{\partial\varphi}{\partial r}=-\frac{\mathrm{A}\kappa\sin(\sigma t-\kappa r)}{r}+\frac{\mathrm{A}}{r^2}\cos(\sigma t-\kappa r).$$

For values of $\kappa>1$, then, as r increases the second term becomes less and less important, and the velocity falls off as $1/r$.

Let w_1 be the velocity of a particle at a distance, r_1, at time t, and w_2 be the velocity of a particle at a distance, r_2, at time t.

Then $$w_1=\mathrm{B}\,.\,\frac{\sin(\sigma t-kr_1)}{r_1}$$

and $$w_2=\mathrm{B}\,.\,\frac{\sin(\sigma t-\kappa r_2)}{r_2},$$

where $\mathrm{B}=-\mathrm{A}\kappa$. The mean value of w_1w_2 over a period T

$$=\frac{\mathrm{B}^2}{\mathrm{T}}\int_0^{\mathrm{T}}\frac{\sin(\sigma t-\kappa r_1)\sin(\sigma t-\kappa r_2)}{r_1r_2}dt$$

$$=\frac{\mathrm{B}^2}{2\mathrm{T}}\int_0^{\mathrm{T}}\left[\frac{\cos\kappa(r_2-r_1)-\cos\{2\sigma t-\kappa(r_1+r_2)\}}{r_1r_2}\right]dt$$

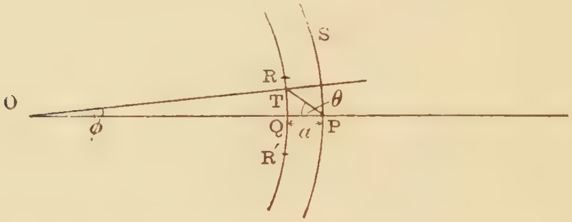
$$=\frac{\mathrm{B}^2}{2}\cdot\frac{\cos\kappa(r_2-r_1)}{r_1r_2},$$

the second integral being zero.

§ 5. We proceed along similar lines to those employed by Dr. Robinson in his discussion of the striations formed in a Kundt's tube.

Assume that a particle in one striation is kept in equilibrium under the forces of repulsion due to the particles of the neighbouring striæ. As the forces vary inversely as the fourth power of the distances between the particles, we only take the nearest striation on either side into account.

Let RQ and PS be portions of two consecutive striæ.



The force of repulsion between a particle at T and one at P is proportional to $\frac{\cos\theta(3-5\cos^2\theta)}{\mathrm{TP}^4}$, and is directed along OP (approximately).

Let n = number of particles per centimetre in the striation RQ. Then the force of repulsion (F) on the particle at P due to the striation RQ

$$-C\int\frac{\cos\theta(3-5\cos^2\theta)}{\mathrm{TP}^4}\cdot rd\varphi\,.\,n,$$

where r=OQ and C=const.

As the force falls off inversely as the fourth power it is unnecessary to take the whole of the striation QR into account, and we get a sufficient approximation by taking a portion, RR′, such that QR=QR′=a, when PQ=a.

Hence φ is a small angle.

The limits of φ are $\pm a/r$.

Expressing θ in terms of φ, and evaluating the integrals, it is easy to show that $\mathrm{F}\propto(\mathrm{C}\,.\,n/a^3)\times$ constant.

Substituting the value of C, viz., $-3\pi\rho a_1{}^3\,.\,a_2{}^3w_1w_2$, we see $\mathrm{F}\propto w_1w_2/a^3$.

[NOTE. The approximations used in calculating the force due to the striation QR amount to the same as regarding the portion near P as being straight: hence, the final value is of the same form as that given by Robinson for straight striæ.]

If we have a number of striations of radii r_1, r_2 . . . r^n, then the force on a particle of the second due to the striation at a distance, r_1,

$$\propto \frac{w_1 w_2}{(r_2-r_1)^3} \propto \frac{\cos\kappa(r_2-r_1)}{r_1 r_2 (r_2-r_1)^3},$$

and the force due to the third striation on the same particle of second

$$\propto \frac{w_2 w_3}{(r_3-r_2)^3} \quad \text{or} \quad \frac{\cos\kappa(r_3-r_2)}{r_2 r_3 (r_3-r_2)^3}$$

For equilibrium we have (assuming n to be the same for all striæ, and that the particles have same radius)

$$\frac{\cos\kappa(r_2-r_1)}{r_1 r_2 (r_2-r_1)^3} = \frac{\cos\kappa(r_3-r_2)}{r_2 r_3 (r_3-r_2)^3}.$$

$$\therefore \qquad \frac{r_3-r_2}{r_2-r_1} = \left(\frac{r_1 r_2}{r_2 r_3}\right)^{\frac{1}{3}} \times \left\{\frac{\cos\kappa(r_3-r_2)}{\cos\kappa(r_2-r_1)}\right\}^{\frac{1}{3}}$$

$$\text{Also} \qquad \frac{r_4-r_3}{r_3-r_2} = \left(\frac{r_2 r_3}{r_3 r_4}\right)^{\frac{1}{3}} \left\{\frac{\cos\kappa(r_4-r_3)}{\cos\kappa(r_3-r_2)}\right\}^{\frac{1}{3}}.$$

$$\text{Finally} \qquad \frac{r_{n+1}-r_n}{r_n-r_{n-1}} = \left(\frac{r_{n-1}\,.\,r_n}{r_n \quad r_{n+1}}\right)^{\frac{1}{3}} \frac{\cos\kappa(r_{n+1}-r_n)}{\cos\kappa(r_n-r_{n-1})}.$$

$$\therefore \qquad \frac{r_{n+1}-r_n}{r_2-r_1} = \left(\frac{r_1 r_2}{r_n\,.\,r_{n+1}}\right)^{\frac{1}{3}} \frac{\cos\kappa(r_{n+1}-r_n)}{\cos\kappa(r_2-r_1)}$$

$$= \left(\frac{r_1}{r_{n+1}}\right)^{\frac{2}{3}} \frac{\cos\kappa(r_{n+1}-r_n)}{\cos\kappa(r_2-r_1)} \text{ approx.}$$

With regard to the cosine term, the following considerations show that it is sensibly unity :—

$$\kappa = \frac{2\pi}{\lambda}. \quad \text{For } \lambda = 1 \text{ cm. approx. } \kappa = 2\pi.$$

Taking average values for the striæ distances close in and far out we may put

$$r_2 - r_1 = \frac{10}{200} \text{ cm. and } r_{n+1} - r_n = \frac{6}{200} \text{ cm.,}$$

whence we have

$$\frac{\cos 2\pi\,.\,\frac{6}{200}}{\cos 2\pi\,.\,\frac{10}{200}} = \frac{\cos 11°}{\cos 18°} = \frac{0{\cdot}982}{0{\cdot}95}.$$

Hence, approximately,

$$\frac{r_{n+1}-r_n}{r_2-r_1} = \left(\frac{r_1}{r_{n+1}}\right)^{\frac{2}{3}}.$$

Experimental Details.

§ 6. In most of the experiments an induction coil was used and various condensers inserted in the discharge circuit. The striations were measured by a travelling microscope fitted with a micrometer eyepiece scale (20 divisions per millimetre), and, where not otherwise stated, the striæ distances are given in terms of the micrometer divisions. It is remarkable how well on the whole the striæ lend themselves to measurement, although irregularities are to be noted in places where the striæ are much forked, as a fork modifies the neighbouring striæ considerably. The number visible in the eyepiece was noted and divided into the total distance on the scale. The value obtained was taken as the average distance apart for the distance from the centre given by the travelling microscope. The average distance between the striæ was obtained every few millimetres, and as far out from the centre as possible.

In comparing the results obtained with those given by the theory some care is necessary. If, for example, the intervals between consecutive striæ for different values of r are compared with the interval for a particular value of r, *i.e.*, if in the notation here used the striæ intervals are compared with the striæ interval r_2-r_1 at a distance r_1 from the spark, then, if there is any irregularity in r_2-r_1 due to the presence of forks in the neighbourhood, this error will be communicated to all. The results are, therefore, worked out in two ways. First, the value of the striæ interval at a place where the striations are well formed is taken as origin, and all others referred to it. Secondly, the results are taken in pairs 1, 2 ; 2, 3 ; . . .

In Table I. a set of values obtained is given :—

Table I.

r cms.	Mean striæ distance.	Observed values.	Calculated values.	Observed values.	Calculated values.
		$\frac{r_{n+1}-r_n}{r_2-r_1}$	$\sqrt[3]{\left(\frac{r_1}{r_{n+1}}\right)^2}$	$\frac{r_{n+1}-r_n}{r_2-r_1}$	$\sqrt[3]{\left(\frac{r_1}{r_{n+1}}\right)^2}$
7·1	6·0	...	...		
6·8	6·25	0·96	0·97	0·96	0·97
6·32	6·88	0·87	0·93	0·91	0·95
5·88	6·63	0·91	0·88	0·94	0·91
5·4	7·06	0·85	0·83	0·94	0·95
5·0	7·38	0·81	0·79	0·96	0·95
4·7	7·8	0·77	0·76	0·95	0·96
4·03	8·3	0·72	0·69	0·94	0·90
3·66	10·0	0·60	0·64	0·83	0·94
3·27	10·8	0·56	0·60	0·93	0·93
2·74	11·1	0·54	0·53	0·97	0·89

In columns 3 and 4 of the table $r=7{\cdot}1$ is chosen as the distance with which all the others are compared. In columns 5 and 6 the results are compared pair by pair.

We give below another set of values for a different experiment :—

TABLE II.

cms.	Mean striæ distance.	Observed. $\frac{r_{n+1}-r_n}{r_2-r_1}$	Calculated. $\sqrt[3]{\left(\frac{r_1}{r_{n+1}}\right)^2}$	Observed. $\frac{r_{n+1}-r_n}{r_2-r_1}$	Calculated. $\sqrt[3]{\left(\frac{r_1}{r_{n+1}}\right)^2}$	Observed. $\frac{r_{n+1}-r_n}{r_2-r_1}$	Calculated. $\sqrt[3]{\left(\frac{r_1}{r_{n+1}}\right)^2}$
3·41	12·5	0·39	0·46	1·67	1·39	0·77	0·82
4·63	9·6	0·51	0·57	1·28	1·13	0·78	0·88
5·6	7·5	0·65	0·64	...	...	0·95	0·93
6·29	7·1	0·69	0·69	0·95	0·93	0·86	0·94
6·95	6·13	0·80	0·74	0·82	0·87	0·92	0·88
8·43	5·65	0·86	0·84	0·75	0·76	1·00	0·97
8·85	5·65	0·86	0·87	0·75	0·74	0·92	0·96
9·41	5·2	0·94	0·91	0·69	0·71	0·96	0·97
9·91	5·0	0·98	0·94	0·67	0·68	0·98	0·94
·9	4·88	...	...	0·65	0·64		

In columns 3 and 4 $r=10·9$ is chosen as origin. In columns 5 and 6 $r=5·6$ is chosen as origin. In columns 7 and 8 the readings are taken pair by pair.

The preceding tables show that there is, on the whole, a good agreement between the experimental values and those calculated from the theory. The differences are of the same order as the errors of observation. A better agreement could not well be expected, for the wave given by the spark is only approximately spherical in type. Moreover, there is some uncertainty in determining the striæ interval where the striæ are forked or badly formed ; again, the action of the nearest striæ only has been taken into account in the theory, and viscosity and other frictional forces have been left out of consideration.

With particles of comparatively large dimensions, such as sand or emery powder, the striæ are in the form of chains of particles placed end to end. A photograph of sand striations, magnified 20 diameters (*see* Fig. 1, Plate) shows very clearly how the particles arrange themselves in line due to the attraction between them in a direction perpendicular to that of propagation of the wave.

If the spark be horizontal and just above the surface of the plate, the striations extend only a short distance in the direction of the spark-gap, but much farther in a perpendicular direction (Fig. 2). Circular arcs with centre in the mid point of the spark-gap have been struck, and it will be noticed that the striations are practically concentric with them.

Experiments on Striation Formation in Channels.

§ 7. The investigation in § 5 shows that the force of repulsion between two striations varies as w_1w_2/a^3. Now, for a plane wave the amplitude of the particle velocity is independent of the distance from the source, and we should expect the striæ distance to be constant also. If the wave be confined by parallel walls, continued reflections at the walls will render the wave approximately plane, and hence the striæ distance should be approximately constant. To test this, channels were formed in the plate by laying strips of glass or wood parallel to each other at various distances apart. The striations formed, especially those in the narrow channels, were very well defined and extended much farther from the source than those formed by the same spark when no channels were present. A set of values are given below. The spark was the same length throughout, and in the first three cases the tops of the channels were left open.

Channel width.	Striæ distance.
2·5 cm.	6·9
1·5 cm.	10·3
0·9 cm.	12·3
1·65 cm. covered channel	11·1

In the case of the narrowest channel the spark was passed for a considerable time. The above results are the mean values of the striæ distances over the whole length of the channel. The striæ distance was not constant, but fell off slowly as the distance from the spark increased. It was found that, other conditions being the same, the striæ were farther apart in covered channels than in open ones of the same width.

The experiments made with channels of different widths and with rectangular tubes gave the same results as those made by Richmond for tubes of circular section. For the same spark the striæ distance is a maximum for a certain width. With narrow channels the whole of the section is covered by uniform striations, perpendicular to the walls; but for wider channels there is a well-defined central pattern and also side patterns, but in the portions of the channel on either side of the central pattern the striæ are not well formed (Fig. 3). In working with ordinary tubes it is evident that the central pattern only is obtained. The fact that the striæ intervals vary with the

channel and tube is perhaps to be ascribed to the reflections which take place at the walls. These will reinforce one another at the centre if the spark be opposite the middle of the channel, and the extent of reinforcement will vary with the size of the tube.

Experiments have been made with ordinary tubes with the spark-gap in the first place opposite the end of the tube (as in Richmond's experiments). Secondly, T pieces were blown on the tube and the terminals introduced through these into the tube and sealed with wax. It was found that the character of the striations depended upon whether the ends were open or closed. In one tube, for example, one end of which was sealed off a short distance from the T piece, the motion was very energetic, and during the passage of the spark the powder gathered itself up in little clouds about 1 cm. apart. On closing the open end with the hand or a cork the striæ were much closer together, and were similar to those obtained when the spark-gap was at the end of an open tube. It seems probable that stationary wave motion is set up with certain end conditions, and the powder gathers itself up into clouds at the nodes. The wave-length of this disturbance, calculated from the nodal distances, would agree very well with the value ascribed to the wave-length earlier in the Paper. It was found that the striæ distance falls off slowly as the distance from the spark increases, which agrees with the experiments with channels. This decrease is without doubt due to the diminution in the amplitude owing to viscosity, &c.

Experiments with Different Obstacles in the Path of the Wave.

§ 8. Some interesting effects have been obtained by placing various obstacles on the plate near the spark-gap. With a plane obstacle formed of glass or wood strips laid across the plate the striations near the central portion of the obstacle are parallel to the "wall." At the sides short striæ perpendicular to the wall are obtained (Fig. 4). The striæ distance falls off in going outwards from the spark, but becomes much greater close to the wall. This is doubtless caused by the increase in amplitude of the disturbance close to the reflector owing to the superposition of incident and reflected waves. Farther from the centre of the obstacle the incident and

reflected waves give rise to a wave motion along the face of the obstacle and produce perpendicular striæ. The formation of striations in this way provides an excellent means of illustrating the reflection and interference of waves diverging from a small source. Where two waves cross one another we have a "tracery" pattern, and where they reinforce well-defined striations. If the spark-gap is arranged at one focus of an ellipse, the converging of the striations round the second focus and the heaping up of the powder there is very apparent. Fig. 4 illustrates the bending of the waves round small obstacles. They illustrate the diffraction of aerial waves of short wave-length. The pattern between the small cylinders closely resembles the lines of force between two electrically charged cylinders or magnet poles, and affords an interesting illustration in support of the analogy between the hydrodynamical and magnetic forces.

The striations obtained in a channel formed by walls making an angle with each other are worthy of some remarks. When the spark is opposite the middle of the wide end, then a well-defined central pattern is formed slightly concave to the spark. At the sides perpendicular striæ are formed, while in between striations parallel to the sides are formed (Fig. 3). The central pattern is formed by reinforcement along the centre line of the wave reflected from the sides.

Regular polygons were formed by glass strips, the spark-gap being at the centre of the figure. Passing from the spark to each angular point and to the centre of each side a narrow strip of striations was obtained. Around the sides of the polygon short striations perpendicular to the sides are formed (*see* Fig. 5).

§ 9. The striations from which measurements were taken were formed in lycopodium. Cork dust gives clear and steep striations very suitable for photography. Amorphous silica gives ill-defined striations, apparently due to a cloud of the fine powder settling on them. Sand and other coarse-grained powders form chains of single particles which do not extend outwards for such great distances as do those produced with smaller particles. By using sand in a narrow channel closed at the end and focusing a low-power microscope upon the grains the formation of the striations can be watched. It is interesting to see a grain move up to a striation and take its place in the chain, and the repulsive force between particles in neighbouring chains was clearly indicated.

Fig. 1.—Photomicrograph of Sand Grains.

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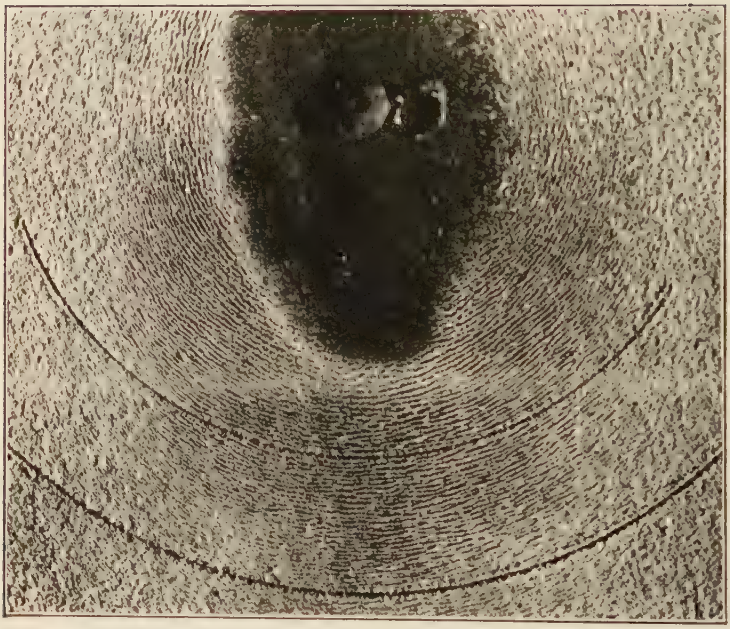

FIG. 2. STRIATIONS WITH CIRCULAR ARCS.

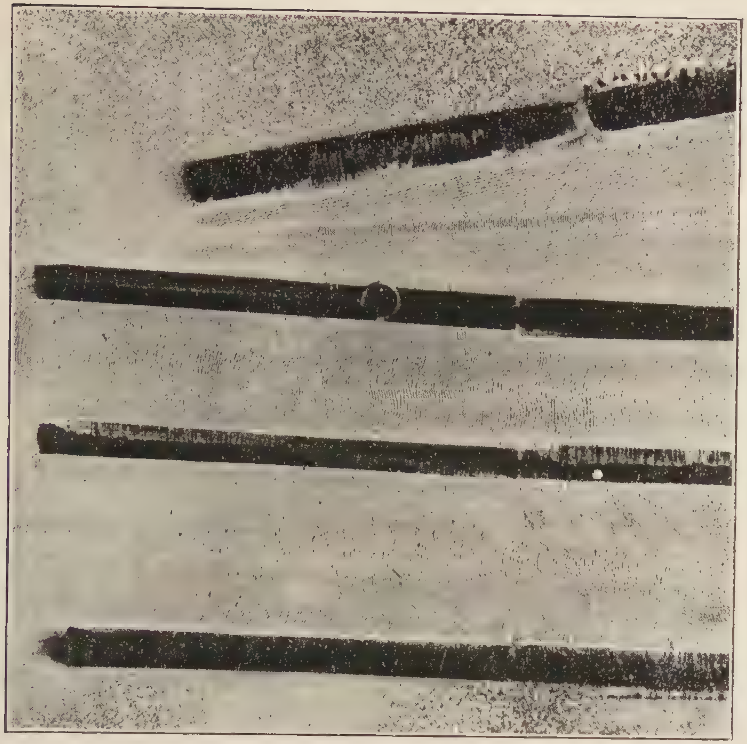

FIG. 3.—STRIATIONS IN LYCOPODIUM—CHANNELS.

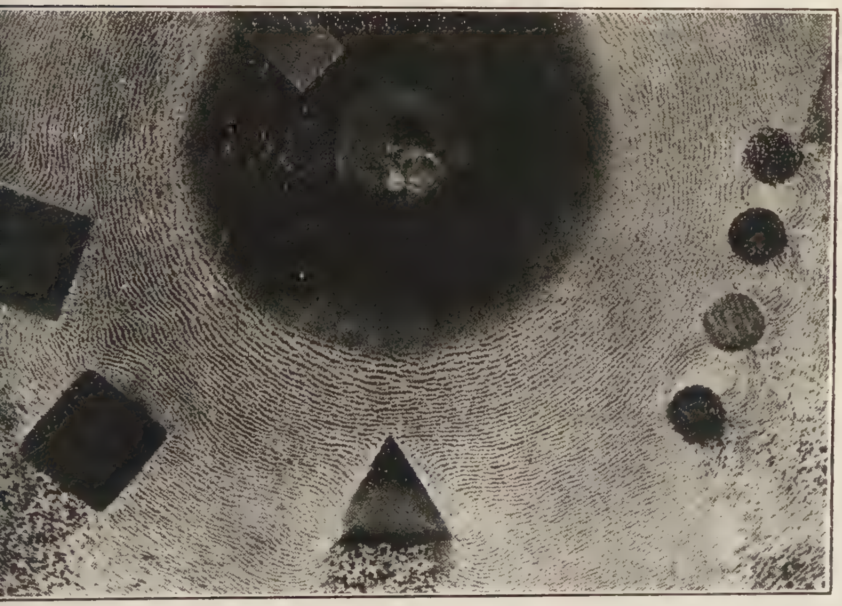

FIG. 4.—STRIATIONS IN CORK SMALL OBSTACLES.

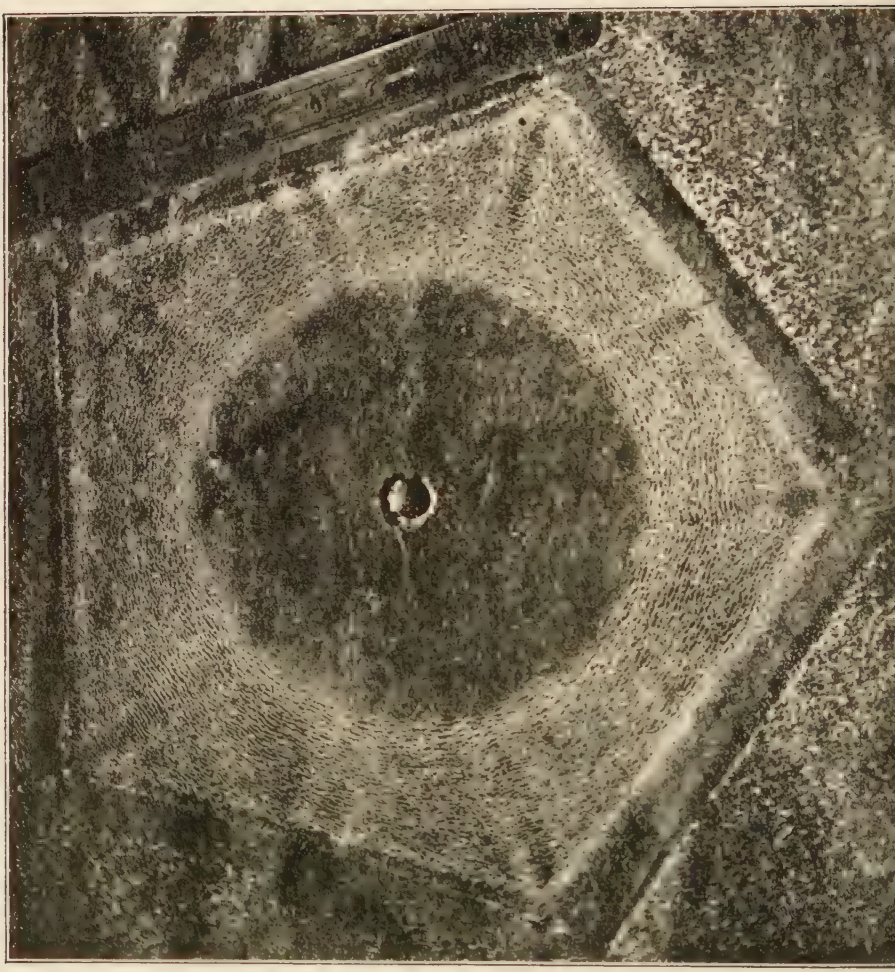

Fig. 5.—Striations in Cork.—Closed Polygon (Pentagon).

The form of the cross-section of the striations was obtained in the following manner :—

A plate was cut into two pieces along a line passing through the central hole. These pieces were carefully fitted together, being laid on another clean plate, so that the edges were flush. Striations were formed in the ordinary way, and the two parts carefully separated and viewed by a microscope of moderate power. The sides of the striations were fairly steep (with cork dust they are nearly vertical) and their height about equal to the distance between them.

[NOTE.—During the writing up of this Paper a short note by Dr. Robinson appeared in the "Phil. Mag.," for February, in which the suggestion was made that the striæ produced by a spark are of the same nature as those obtained in a Kundts tube.]

We desire to express our best thanks to Mr. Thomson, M.A., B.Sc., for his valued criticism and interest in the research.

[NOTE, APRIL 6TH.—Our attention has been drawn to a Paper by Dr. Cook in the "Proceedings" of the Society (1888), illustrating the formation of dust striations on a plate by a horizontal spark. In a footnote Cook refers to an experiment due to Guthrie in which an elliptical reflector was used and a spark arranged at one focus, whereby striations similar to those shown in Fig. 5 were obtained.]

ABSTRACT.

The formation of dust striations by electric spark has been investigated by many observers. The Paper attempts to explain their formation as being due to hydrodynamic forces existing between the dust particles while the wave motion is passing over them. The application of this theory to the striations in a Kundts tube has been made by Kœnig and Robinson. The wave motion is assumed to be of the spherical progressive type and expressions are obtained for the intervals between consecutive striæ and the distances of the striæ from origin. Measurements were made of striæ formed on a glass plate with vertical central spark. The agreement between theory and experiment is within the experimental error. Experiments with channels of various shapes were made. Illustrations of the various striæ patterns obtained with small obstacles and reflecting surfaces are given, and the use of these as a convenient means of indicating reflection interference and diffraction of sound waves is pointed out.

DISCUSSION.

Mr. A. CAMPBELL thought the authors' results justified their theory. In the experiments recently described by himself and Mr. Dye there was fair correspondence between the actual oscillation frequency and the closeness of the dust streaks. The frequencies used (50,000 up to 700,000 $\sim$ per second)

were very much smaller than those used by the authors (100 million per second), while the damping was not considerable, at least 25 to 30 oscillations taking place in each spark train. Thus it might be expected that very different effects would occur in the two cases.

Dr. RUSSELL said that the formation of striations was one of the best methods of telling whether a spark was oscillatory and of high frequency or not. All that was necessary was to sprinkle a little lycopodium or other light powder on a glass plate or on the surface of water and hold it near the spark. If the spark were oscillatory, circular striations would be at once formed. He suggested that the method might be used as a rough test for oscillations in the same way that chemists use litmus as a test for acids. He had carried out experiments on the striations formed in long glass tubes of various diameters. The distances between the striations varied with the frequency of the spark. He was struck by the fact that the distance between them was, except near the ends, practically constant along the whole length of the tube. Closing one end of the tube or varying the length had very little effect on this distance. The striations were always in motion, and the complete theory must be very complex. He thought that the best way to produce oscillatory sparks for these experiments was to use an induction coil with two Leyden jars connected across its secondary terminals in Lodge's manner. He congratulated the authors on the interesting theoretical results they had obtained.

Dr. MARSH, in reply, stated that the experimental results of Richmond and the authors showed that there was no relation between the striæ intervals and oscillation frequencies. He referred to a note in the "Philosophical Magazine" for February by Dr. Robinson, who performed a series of experiments in which the frequency was kept constant, but the intensity of the spark was varied. The results showed that the striæ intervals *increased* with the intensity of the spark, so proving that the striations are not formed at the nodes of a secondary wave motion set up in the tube. In the authors' experiments the striæ were formed by progressive waves, and this shows clearly that the explanation must be looked for in another direction. The frequency was varied within wide limits, but no marked change was observed in the striæ formed on the plate. In long tubes they found that the striæ intervals became slightly smaller with increasing distance from the spark end.

XXVIII. *The Method of Constant Rate of Change of Flux as a Standard for Determining Magnetisation Curves of Iron.* *By* J. T. Morris *and* T. H. Langford, *B.Sc.*

Communicated by Prof. C. H. Lees, F.R.S. Received March 22, 1911. Read May 12, 1911.

Introduction.

Doubts must have arisen in the minds of many investigators as to how nearly the magnetisation curves of a sample of iron obtained by differing methods would agree with one another.

Discrepancies which have been found from time to time may be attributed to one or more of the following causes :—

(i.) Inaccuracies in experimenting.

(ii.) Lack of exact repetition of previous conditions. In other words, the physical state may differ in different experiments with regard either to hardness, temperature or previous magnetic treatment.

(iii.) Differences inherent in the methods employed.

The present research was instituted with the object of studying the last of these three causes, and obtaining definite information as to the magnitude of these differences (*a*) between the magnetisation curves for a given sample of iron when determined by the older methods, in which the flux is changed suddenly, and (*b*) by a method in which it is changed slowly and at a uniform rate.

Methods in General Use.—The relation between the magnetising force H and the flux-density B produced by it in a specimen of magnetic material is usually determined by one of the following methods :—

(*a*) *Ballistic Methods.*—The specimen, preferably in the form of a closed ring, is wound with primary and secondary coils, and the change of flux, caused by a sudden change of current in the primary coil, is measured by the extent of the first swing given by the moving system of a ballistic galvanometer in series with the secondary winding.

(*b*) *Alternate-current Methods.*—The specimen is prepared as in (*a*) with primary and secondary coils, and alternating currents of commercial frequency are passed through the primary winding, the maximum currents and the corresponding maximum values of the flux-density being determined.

(c) *The "Fluxmeter" Method.*—The Grassot fluxmeter* is an instrument somewhat similar to a ballistic galvanometer, but the moving coil is suspended by a single cocoon fibre exerting no appreciable controlling torque. The movable coil is connected to the secondary winding of a specimen ring, as in the ballistic method, and it is deflected through an angle which is proportional to the time-integral of the voltage induced in the secondary winding. The deflection is thus proportional to the total change of flux through the ring, and is independent of the time occupied by the change.

(d) *Magnetometric Methods.*—A long thin bar of the material to be tested is wound with a magnetising coil, and the strength of pole produced by a known number of ampere-turns is determined by means of a magnetometer.

(e) *Attraction Permeameter Methods.*—The sample is arranged to close the magnetic circuit of an iron yoke of such dimensions that its reluctance may be neglected; then the mechanical force of attraction between the specimen and the yoke is measured by observing either the force necessary to detach the specimen or the increase of the force required to cause the specimen to slide on the yoke.†

Comparison of Methods.

Permeameter measurements have the advantage of rapidity, but the accuracy attainable is not very high, owing to the indefinite degree of contact between the specimen and the yoke. This method is generally employed only for workshop tests. The magnetometric method is liable to inaccuracies due to the lack of uniformity in the magnetisation of the sample, including the indeterminate position of the poles and their demagnetising action. Alternate-current methods have the disadvantage that they are indirect, the required values of the current and flux-density having to be deduced from the observations by a complicated method if accuracy is to be ensured. When the greatest accuracy is required, ballistic methods on ring samples are, therefore, those to which resort is usually made. The test is commonly made by one of two methods: In one of these the current through the primary

* "Some New Electrical Instruments," The Electrician, Vol. LVI., p. 560, 1906.

† S. P. Thompson, "The Electromagnet"; W. H. F. Murdoch, "The Magnetic Testing of Iron," I.E.E., Vol. XL., p. 137, 1907.

winding is increased up to a certain value and then suddenly reversed to obtain a reading; in the other method the current is increased or diminished by a step at a time, and the consequent ballistic reading taken. This difference in the character of the magnetic change to which the iron is subjected is liable to cause differences in the results obtained, partly owing to magnetic "creep" or viscosity.

A method proposed by C. F. Scott, and apparently first described by J. S. Peck,* as applied to large transformers, possesses the marked advantage over ballistic methods that the variation of the flux takes place at a known and constant rate, enabling experiments to be repeated under exactly similar conditions. Rates of change varying from 50 to 250 lines per square centimetre per second were actually employed in the experiments. Further, standardisation can be more readily performed, as a mutual induction is not required. This method has been modified by Dr. D. K. Morris so as to render it practicable for tests on small samples of iron.†

The Method of "Constant Rate of Change of Flux."—Consider an iron ring in which a flux, N, is produced by a current, C, flowing in a primary winding. By varying the current C in a continuous manner the flux N in the ring is continuously varied, and at any instant the E.M.F. E induced in a secondary winding is proportional to the rate of change of the flux N—*i.e.*, to dN/dt—and the total change of flux which has taken place in the ring during any interval of time $(t_2—t_1)$ is $N_2—N_1$ $=\int_{t_1}^{t_2}\frac{dN}{dt}\cdot dt.$

If now the primary current C be varied at such a rate that the induced E.M.F. E is maintained at a constant value, the flux N is known to be varying at a constant rate, $dN/dt=k$.

In this case the total change of flux through the ring is simply proportional to the time during which it has taken place, for

$$\int_{t_1}^{t_2}\frac{dN}{dt}\cdot dt=k\int_{t_1}^{t_2}dt=k(t_2—t_1).$$

If n_2 be the number of turns on the secondary winding, and

* J. S. Peck, "On Testing Large Transformers," "Electrical World and Engineer," Vol. XXXVII., p. 1083, 1901.

† D. K. Morris and G. A. Lister, "Transformers and Transformer Iron," I.E.E., Vol. XXXVII., pp. 282-294.

E be the value of the constant E.M.F. in volts induced therein, we have

$$E = dN/dt \times n_2 \div 10^8 = kn_2/10^8$$

$$\therefore \quad k = \frac{E \times 10^8}{n_2},$$

and total change of flux

$$(N_2 - N_1) = k(t_2 - t_1) = \frac{E \times 10^8}{n_2}(t_2 - t_1).$$

Hence a magnetisation curve can be deduced from readings of current and time when the current is varied at such a rate that the secondary induced E.M.F. is constant and of known value.

Since the rate at which the flux is varied can be made as slow as desired, any "creep" in the magnetisation due to magnetic viscosity will be recorded. This method, therefore, appears to be preferable to others as the standard method for determining the fundamental magnetic properties of iron.

The following experiments were undertaken to investigate the differences, if any, between the results obtained by this and by other methods :—

Methods Experimentally Examined.

I. *Method of Constant Rate of Change of Flux, or, more shortly, that of "Uniformly-varying Flux"* (or "Constant Induced Voltage," as it has been termed by previous writers).—After the iron has been demagnetised, the flux-density is continuously raised from zero to the maximum value, as above described.

II. "*Slow Cyclic*" *Hysteresis Loops.*—A similar method to the first, except that the iron is carried through cycles of magnetisation of gradually increasing maximum value, and the maximum values of B and H are plotted to give the magnetisation curve.

III. "*Step-by-step*" *Magnetisation Curve.*—The iron is first demagnetised and the current is then raised by small increments, the flux-density produced being deduced from the throws observed on a ballistic galvanometer after each increment of current.

IV. "*Step-by-step*" *Hysteresis Loops.*—A similar method to the last, except that the iron is carried through cycles of magnetisation of gradually increasing maximum value, and the

maximum values of B and H are then plotted to give the magnetisation curve.

V. *Method of Reversals.*—The iron is demagnetised, the current raised to a certain value, reversed, and the resulting throw taken on a ballistic galvanometer. This is repeated for currents of gradually increasing value, and the magnetisation curve so obtained plotted.

VI. *Alternating-current Magnetisation Curve.*—The magnetisation curve is deduced from readings of the magnetising current and induced E.M.F. when the winding of the ring is traversed by an alternating current at a frequency of 60 cycles per second.

Before each reading recorded by methods II., IV. and V. the iron was brought into a cyclic condition by carrying it through a sufficient number of cycles of magnetising force of the same maximum value, some 50 cycles being generally sufficient for this purpose.

Practical Details of Method of Uniformly varying Flux.

In Fig. 1 is given a diagram of connections. The voltage induced in the secondary winding S is balanced by the fall of potential over a known resistance, r_1, in series with a known resistance, r_2, across a cell, B, of known E.M.F., equality being indicated by a zero reading on a galvanometer, G. The battery B is closed on to the circuit r_1r_2, and simultaneously the primary current is caused to begin to change by means of the rheostat A, and the experimenter has continuously to adjust the rheostat at such a pace as will maintain the galvanometer reading at zero. This adjustment requires some skill, but an average experimenter easily acquires this with a little practice.

In making a hysteresis test the primary current is continuously varied from a maximum positive value to an equal negative value by a specially designed resistance,* in which two sliders, a, b, connected to the primary winding P, are moved in opposite directions across the edges of the zigzag strip resistance A. A photograph of the rheostat employed is shown in Fig. 2.

The primary current is measured on a potentiometer by the

* J. T. Morris, R. M. Ellis and F. Stroude, "The Design of a Continuously Adjustable Rheostat," The Electrician, Vol. LXI., p. 400, 1908.

voltage drop across a known resistance, r_3, the exact times at which the current has certain convenient values being recorded by a chronographic arrangement.

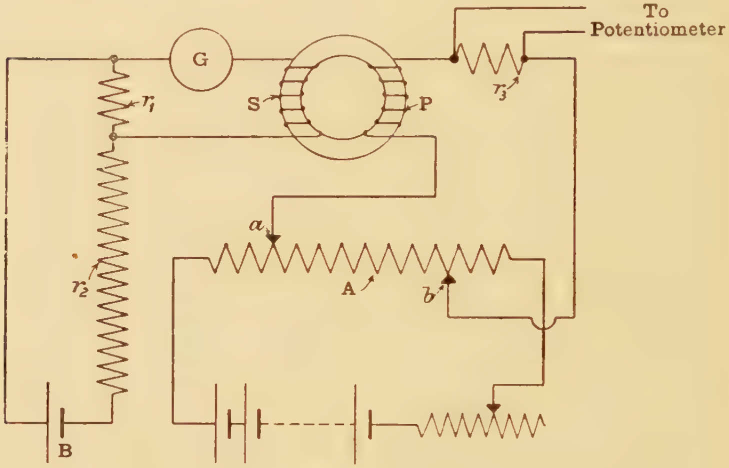

FIG. 1.— CONNECTIONS FOR METHOD OF UNIFORMLY VARYING FLUX.

A Morse inker was employed for this purpose, the receiving coil being in series with a contact made at each swing of a

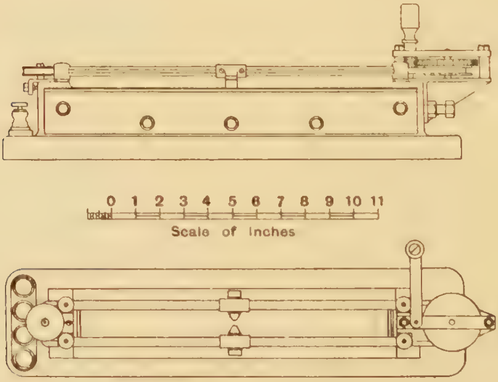

FIG. 2.

pendulum. The arm carrying the inking wheel was arranged to deflect a pencil (which rested on the travelling paper tape) a small distance laterally. A key was also arranged so that, on

depression, the same pencil was deflected laterally double the distance. Timing and observation marks were easily distinguishable. This arrangement was found to be quite satisfactory.

The above electrical connections could be simplified if the galvanometer were directly closed on to the terminals of the secondary winding, but this arrangement has some objections—(1) that it is difficult to allow for the uncertainty in the value of the secondary E.M.F. induced during the time that the galvanometer deflection is attaining its normal magnitude; and (2) that a separate operation has to be performed in order to standardise the galvanometer. A further slight objection is that there is an opposing magnetomotive force due to the current flowing in the secondary winding. This, in general, however, is so small as to be negligible.

Magnetisation Curve by Alternating Current.

It is important in connection with transformers and other alternating-current apparatus to determine the maximum value of the alternating flux-wave produced in iron by a magnetising current wave of known maximum value.

In order to carry out this determination the following experiments were made. The iron ring was demagnetised by a continuously decreasing alternating current from a generator, G (Fig. 3). This current was then gradually increased, its R.M.S. value being read on an ammeter, A, and the R.M.S. value of the voltage induced in the secondary winding S measured by the deflection produced on a Duddell thermo-galvanometer, TG, in series with a known variable resistance, R_2. The thermo-galvanometer was calibrated by a secondary cell, B, of known E.M.F. To determine the wave forms of the magnetising current and secondary induced voltage the strips CS and VS of a Duddell oscillograph were connected as shown, the current strip CS being shunted by a variable portion of a resistance, R_1, and the volt strip VS being connected so as to measure the volts across the primary winding P, the ammeter A and the current strip CS in parallel with its shunt. The resistance of this circuit was measured, and the measured volt wave corrected by subtraction of the corresponding $C \times R$ drop, thus giving the true induced E.M.F. wave. The area of this was then measured by a planimeter, and from this the maximum flux-density was calculated, corresponding to the R.M.S. volts measured on the thermo-galvanometer. This determina-

tion was made for a number of different values of the secondary induced voltage. As a check, the R.M.S. value of the corrected volt wave was determined, and compared with the value obtained by measurement with the thermo-galvanometer on

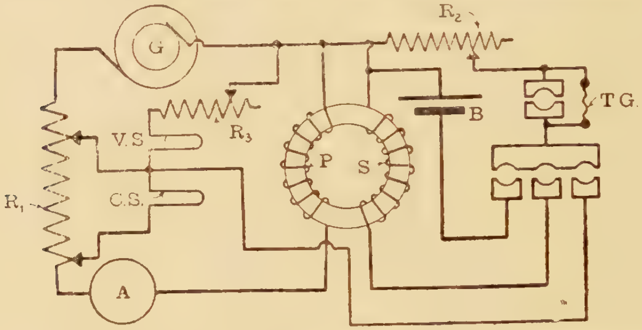

Fig. 3.—Connections for Magnetisation Curve by Alternating Current.

the secondary; also the R.M.S. value of the volts measured by the volt strip VS was read on the thermo-galvanometer and compared with the value obtained from the oscillogram.

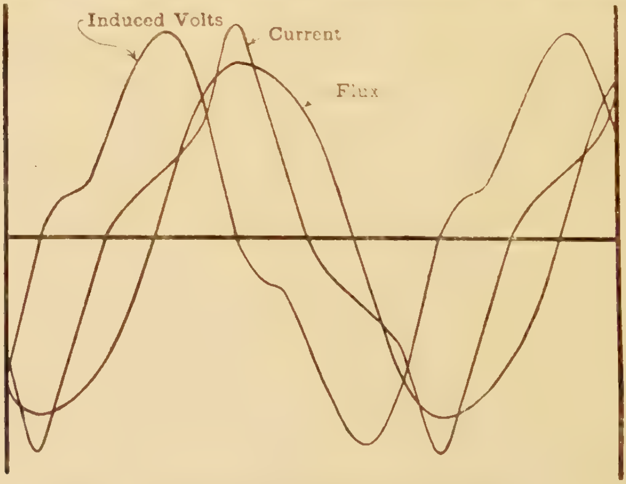

Fig. 4.—Flux Wave Deduced from Oscillogram.

A specimen oscillogram, showing current and induced voltage waves, together with the flux wave deduced therefrom, is shown in Fig. 4.

The value of the maximum flux-density induced may be determined from the R.M.S. value of the induced voltage by the formula

$$\mathsf{B}=\frac{\mathrm{E}}{\mathrm{Z}\times n\times \mathrm{A}}\times k\times 10^{8},$$

where E=R.M.S. value of induced voltage.
Z=number of secondary turns.
n=frequency in cycles per second.
A=cross-sectional area of iron ring in square centimetres.
B=maximum flux-density in lines per square centimetre.
k=factor dependent on the wave-form, being 1/4·44 =0·2252 for a sine wave of induced voltage.

The values of the factor k were experimentally determined at various points on the magnetisation curve, and are given in Table I.

TABLE I.—*Values of Factor "k" in Formula* $\mathsf{B}=\frac{\mathrm{E}\times 10^{8}}{\mathrm{Z}n\mathrm{A}}\times k$.

R.M.S. volts generated per turn at 1 period per second.	Factor "k."
0·000129	0·227
0·000422	0·228
0·00133	0·211
0·00134	0·217
0·00166	0·192
0·00171	0·193

The maximum value of the magnetising current is obtained from its R.M.S. value as measured on the ammeter by multiplying this figure by the form-factor, the experimentally-determined values of which are collected in Table II.

TABLE II.—*Current Wave Form Factors.*

R.M.S. amperes.	Form Factor.
0·194	1·430
0·405	1·370
1·854	1·904
2·57	1·935
5·05	1·820
8·135	2·070

Data of Specimen used.—The ring was made up of 24 stampings, 6 in. external and 5 in. internal diameter, of Sankey's best transformer iron, manufactured about the year 1900, of

24 S.W.G. The mean magnetic length of the ring was found by measurement to be 43·93 cm. The sectional area of the ring was determined from its weight and specific gravity, the latter being determined by weighing in air and in carbon tetrachloride. This liquid is preferable for this purpose to water on account of its non-oxidising properties, and also because of its higher specific gravity. The stampings were insulated from one another by thin paper rings, and the complete ring was lapped over with insulating tape and wound with two No. 19 D.C.C. wires laid side by side, giving two windings of 147 turns each. These could be used in series or in parallel as a magnetising winding, or one could be used as a magnetising winding and the other as a secondary. A secondary winding of 50 turns and one of 10 turns were also wound on. Exact data of the stampings are as follow :—

Weight of pack of 24 stampings	= 653·0 grm.
Volume (sp. gr. = 7·805)	= 83·67 c.c.
Mean length of path	= 43·93 cm.
Sectional area	= 1·904 sq. cm.
Effective thickness	= 1·470 cm.
Actual thickness of laminæ + paper insulation	= 1·616 cm.

The paper insulation occupies about 5 per cent. of the total thickness, and the roughness of the iron surface accounts for about 4 per cent.

Experimental Results.

The results obtained experimentally by the above-described methods I. to VI. are given in the form of curves in Figs. 5, 6 and 7, and readings taken from the plotted curves are given in Tables III. and IV.

The magnetisation curves are shown in Fig. 5, the initial parts being shown separately to an enlarged scale of **H**.

In Fig. 6 the results are shown in the form of permeability curves, and in Fig. 7 in the form of reluctivity curves.

In Fig. 8 the differences between the flux-density obtained by method I. and by methods II., III., IV. and V., with the same magnetising force, are plotted to a greatly enlarged scale against **B** as obtained by method I. The divergence of the magnetisation curve obtained by method VI. from the standard curve obtained by method I. is too great to allow of its being shown in Fig. 8, but is apparent in Figs. 5, 6 and 7.

Fig. 9 shows hysteresis loops taken by methods II. and IV., the maximum value of **H** being the same in each case.

The above experiments were carried out in the electrical

TABLE III.

H	Flux-density B by methods :— I.	II.	III.	IV.	V.	VI.
0·3	360	100	120	100	210	140
0·5	610	220	350	200	400	320
0·7	990	500	750	400	750	610
0·9	1,530	1,080	1,300	900	1,230	950
1·2	2,680	2,500	2,500	2,300	2,400	1,620
1·5	3,840	3,700	3,650	3,600	3,620	2,500
2·0	5,450	5,600	5,100	5,250	5,300	4,000
2·5	6,560	6,750	6,300	6,350	6,550	5,400
3·0	7,480	7,700	7,250	7,200	7,400	6,550
3·5	8,250	8,420	8,020	8,020	8,120	7,650
4·0	8,850	9,080	8,700	8,600	8,700	8,500
4·5	9,440	9,600	9,200	9,060	9,300	9,220
5·0	9,880	10,100	9,620	9,500	9,800	9,800
6·0	10,700	10,920	10,410	10,300	10,500	10,650
7·0	11,330	11,550	11,050	11,040	11,250	11,300
8·0	11,920	12,100	11,600	...	11,850	11,820
9·0	12,400	12,600	12,100	...	12,300	12,300
10·0	12,850	12,960	12,520	...	12,750	12,760
12·0	13,580	13,600	13,110	...	13,400	13,460
14·0	14,200	14,100	13,590	...	13,950	14,200
16·0	14,720	14,510	13,950	...	14,380	14,620
18·0	15,110	14,750	14,300	...	14,750	15,100
20·0	15,400	15,100	14,650	...	15,050	15,450
25·0	16,040	15,510	15,440	...	15,600	16,120
30·0	16,500	16,050	16,010	...	15,830	16,420
35·0	16,840	...	...	...	16,180	16,650
50·0	17,320	...	...	...	16,700	17,020
70·0	17,850	...	...	...	17·270	17·120

TABLE IV.

B	Permeability by methods :— I.	II.	III.	IV.	V.	VI.
500	1,210	740	820	660	880	840
1,000	1,430	1,140	1,240	1,070	1,245	1,075
1,500	1,668	1,510	1,580	1,440	1,545	1,300
2,000	1,950	1,810	1,870	1,750	1,820	1,470
2,500	2,175	2,080	2,080	2,015	2,030	1,630
3,000	2,356	2,270	2,270	2,215	2,220	1,755
3,500	2,500	2,420	2,400	2,370	2,380	1,850
4,000	2,660	2,535	2,480	2,490	2,505	1,940
4,500	2,690	2,640	2,520	2,580	2,590	2,000
5,000	2,740	2,730	2,545	2,620	2,650	2,050
5,500	2,740	2,787	2,555	2,620	2,660	2,090
6,000	2,700	2,780	2,550	2,585	2,645	2,130
6,500	2,620	2,740	2,520	2,520	2,620	2,150
7,000	2,565	2,660	2,465	2,440	2,550	2,160
8,000	2,410	2,490	2,295	2,270	2,360	2,156
9,000	2,190	2,255	2,070	2,035	2,120	2,045
10,000	1,930	2,010	1,830	1,810	1,790	1,910
11,000	1,720	1,765	1,590	1,570	1,660	1,720
12,000	1,465	1,510	1,345	...	1,440	1,500
13,000	1,250	1,260	1,100	...	1,210	1,280
14,000	1,060	1,005	860	...	980	1,070
15,000	868	765	660	...	760	880
16,000	650	520	530	...	520	650
17,000	425	...	...	...	280	330

engineering department of the East London College (University of London), and the authors wish to express their best thanks

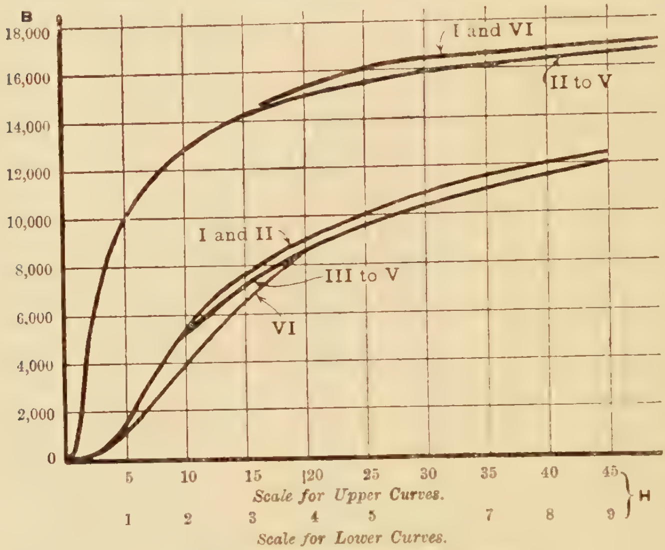

Fig. 5.—Magnetisation Curves.

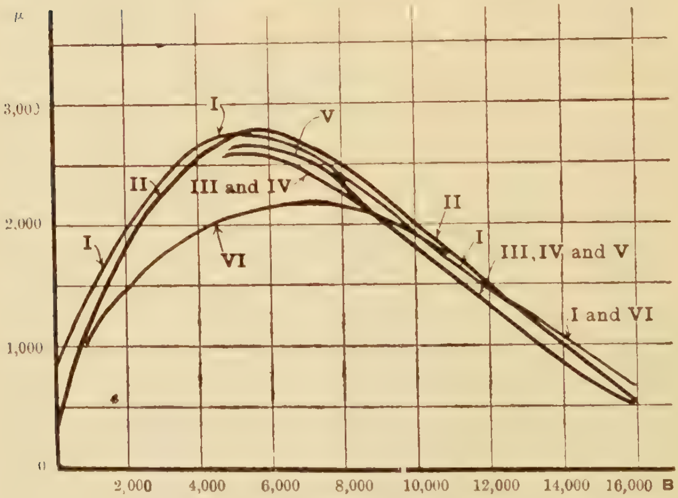

Fig. 6.—Permeability Curves.

for the practical suggestions and assistance in the experimental work given by Mr. F. Stroude, B.Sc.

Discussion of Results.

An examination of the divergences shown in Fig. 8 has led the authors to divide, for purposes of consideration, the curves into three parts: (1) From zero to the approximate point of maximum permeability (B = 5,000); (2) from this point to B = 15,000; (3) above B = 15,000.

1. *Curves below* B = 5,000.—These may be divided into two groups: (*a*) Those in which the magnetisation curve is determined with the flux always changing in the same sense, the iron having previously been completely demagnetised; and

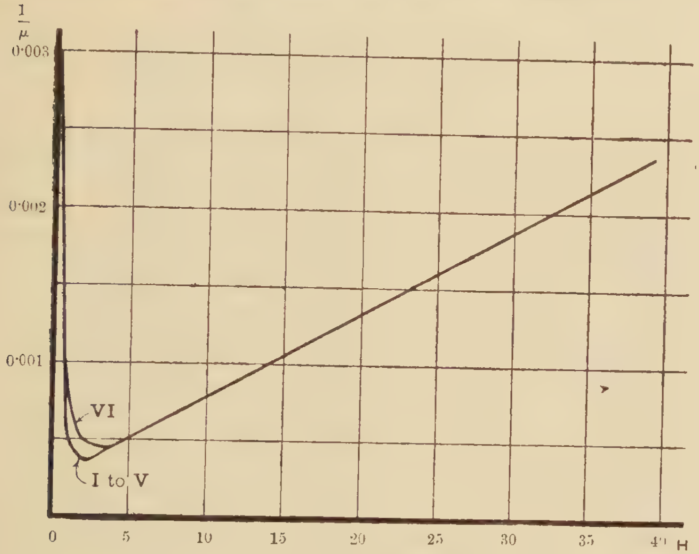

Fig. 7.—Reluctivity Curves.

(*b*) those in which each reading is determined after the iron has been carried through a sufficiently large number of cycles of magnetising force to obtain constancy in the corresponding value of the flux-density.

Considering first those which belong to group (*a*), we have two cases: (i.) That of the magnetisation curve obtained by the uniformly varying flux method (method I.), and (ii.) the magnetisation curve determined by the step-by-step method (method III.).

It will be noted in Fig. 8 that there is a difference between these two curves of, broadly speaking, some 250 lines per square

centimetre, from **B** = 1,000 to **B** = 12,500, and that the step-by-step method is deficient by this amount. This difference appears to be due to some flux not being recorded in the portion of the magnetisation curve below **B** = 500 in the step-by-step method. It is quite probable that this is due to the known behaviour* of iron under weak magnetising forces. When a magnetising force of, say, **H** = 0·2 is suddenly applied, the flux at once changes by a certain amount and afterwards continues to change, though at a diminishing rate; this latter "creep" would not be recorded by an ordinary ballistic galvanometer,

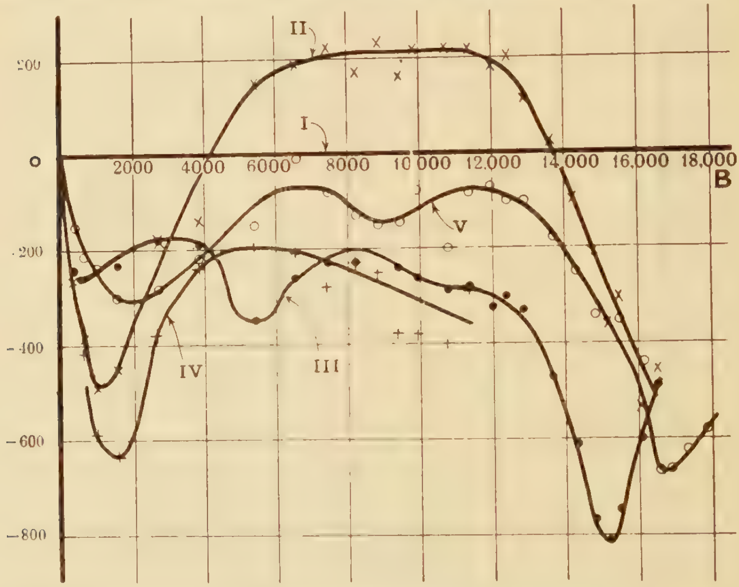

FIG. 8.—CURVE SHOWING DIFFERENCES BETWEEN MAGNETISATION CURVES OBTAINED BY VARIOUS METHODS.

and therefore would not wholly be taken account of in the step-by-step method, but it is included in method I.—that of uniformly varying flux. (The time of one complete swing of the ballistic galvanometer used throughout these experiments was 8·6 seconds.)

With the object of investigating this point the following experiment† was made. With the connections arranged as for

* J. A. Ewing, "Magnetic Induction in Iron and Other Materials," p. 124.
† Hopkinson, "Original Papers," Vol. II., p. 257.

the ballistic test, a sudden change was made in the magnetising current while the circuit of the galvanometer and secondary winding was held open, so that the galvanometer did not receive the "kick" due to the instantaneous change of flux in the ring. The galvanometer circuit was then immediately closed, so that, if there were any residual creeping of the flux, there would be a steady deflection of the galvanometer.

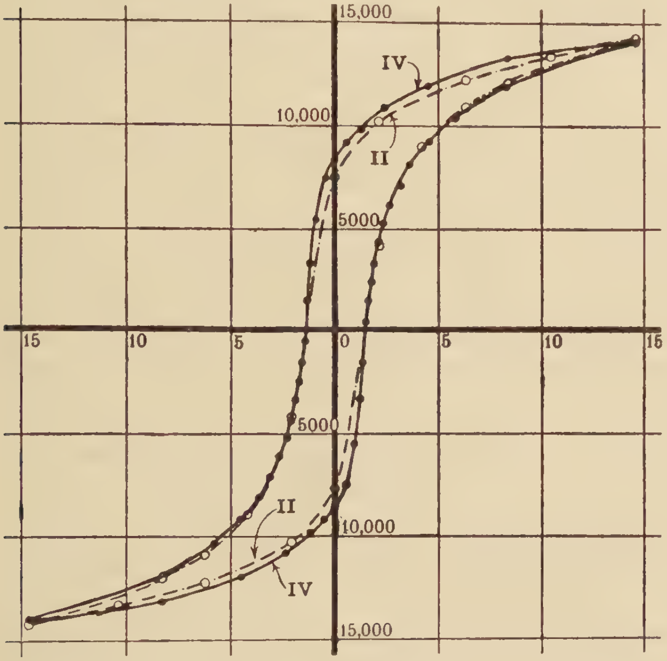

FIG. 9.—HYSTERESIS LOOPS BY METHODS II. AND IV.

The maximum creep observed occurred when a magnetising force of $\mathsf{H}=1{\cdot}25$ in one direction was suddenly changed to the value corresponding to the maximum instantaneous permeability in the other direction ($d\mathsf{B}/d\mathsf{H}=6{,}000$). Under these conditions the creep was 40 lines per square centimetre per second, about 0·1 second after the change in H, and this creep diminished to 3·3 lines per square centimetre per second after about 1 second, disappearing entirely after about 5 seconds. On completely reversing the current the creep after about 0·1 second was only 4·6 lines per square centimetre per

second, and after 1 second it had decreased to less than 1 line per square centimetre per second. This effect, therefore, although appreciable, does not appear to account wholly for the observed deficiency in the ballistic magnetisation curve. It will be noticed that all the curves in Fig. 8 start with a sharp downward inclination from the axis, showing that during the initial part of the magnetisation curve the highest value of **B** for a given value of **H** is attained by the method of uniformly varying flux, or, in other words, a higher value of **H** is required to produce the same observed value of **B** by the other methods. One reason for this may be that when the flux is changed suddenly, as in ballistic methods, an excess magnetising force may be required to counteract the demagnetising force due to eddy-currents caused in the laminations, and this may become considerable when the change of flux is very rapid. In order to investigate this point, an estimate was made of the demagnetising force due to eddy currents, and this was found by calculation to be about $0{\cdot}00000046 \times d\mathbf{B}/dt$. Now the excess **H**, for a flux-density of 500, required for the step-by-step method, as compared with the uniformly varying flux method, amounts to about 0·15, and this amount would be accounted for by eddy currents if the change from $\mathbf{B}=0$ to $\mathbf{B}=500$ took place in about 0·0015 second, which does not seem unlikely. In the case, however, of the magnetisation curve drawn through the tops of the hysteresis loops obtained by the uniformly varying flux method another explanation must be sought, as the rate of change of flux, $d\mathbf{B}/dt$, is too small to produce appreciable eddy currents. It may be that, on slowly increasing the magnetising force from zero to a certain value and then carrying it through cycles having this value for a maximum, the value of **B** obtained initially may be greater than the value obtained after the iron has attained the cyclic state, owing to the first loops being unsymmetrical about the axis of **H**.

With regard to group (*b*), which includes methods II., IV. and V., it is difficult to find any rational explanation for the differences observed. The authors have, however, been impressed with the idea that when a sudden change of considerable magnitude is made in the magnetising force the instantaneous flux variation, as compared with the total flux variation, may depend on the steepness of the magnetisation curve at the point attained instantaneously by the flux; in other words, it might depend not so much on the permeability

at that point as on the rate of change of induction with magnetising force—*i.e.*, on $d\mathsf{B}/d\mathsf{H}$. One is led to imagine that if $d\mathsf{B}/d\mathsf{H}$ is great, then the stability of the molecular magnet grouping is correspondingly weak, and, keeping the Ewing model in view, one would predict that, for a sudden large change in the magnetising force, a magnetic creep depending to a certain extent on this factor $d\mathsf{B}/d\mathsf{H}$ would be expected.

This quantity $d\mathsf{B}/d\mathsf{H}$ and also the permeability, both calculated from the magnetisation curve obtained by the method of uniformly varying flux (method I.), have been plotted in Fig. 10.

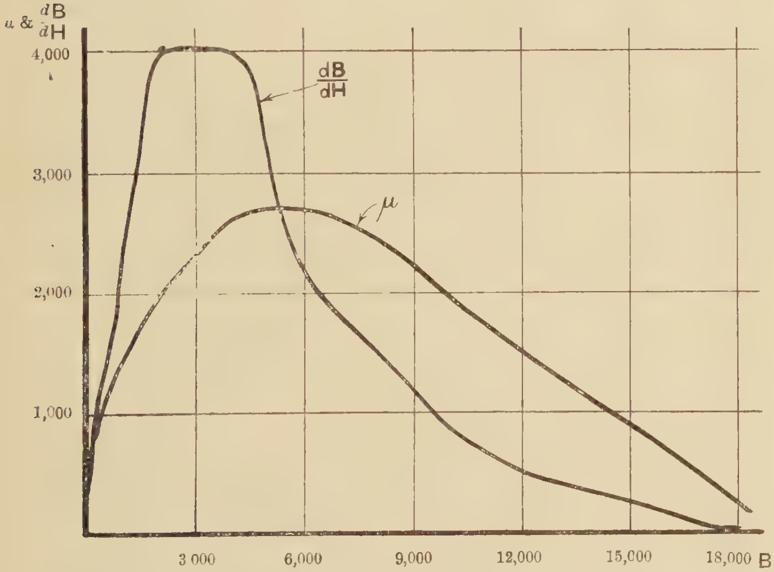

FIG. 10.—μ AND $\frac{d\mathsf{B}}{d\mathsf{H}}$ CURVES.

It will be observed from Fig. 8 that the maximum deficiency in B, as obtained by methods II., IV. and V., occurs at about that part of the magnetisation curve where $\mathsf{B} = 2{,}000$, and Fig. 10 indicates that this is the point where the steepest part of the magnetisation curve commences, or, in other words, where $d\mathsf{B}/d\mathsf{H}$ attains its maximum value. It appears probable, therefore, that the maximum value of B, attained when the iron is carried through a sufficiently large number of cycles of H, depends to some extent upon the slope of the magnetisation

curve at that point, **B** having a diminished value where the magnetisation curve is steepest.*

2. *Curves between* **B** = 5,000 *and* **B** = 15,000.—From Fig. 8 it will be observed that over this range, whilst the magnetisation curve obtained by the step-by-step method (method III.) maintains its deficit of about 200 lines per square centimetre with respect to the magnetisation curve obtained by the proposed standard method of uniformly varying flux (method I.), the magnetisation curves obtained by methods II., IV. and V., in which the iron is brought into a cyclic state before readings are taken, show a decreased deficit with respect to the standard curve, the curve obtained by the slow cyclic loops method (method II.) actually showing, over this range, an excess of some 200 lines per square centimetre. During experiments by methods I. and II. the iron was subjected to the same kind of magnetic change—*i.e.*, the flux was varied slowly and at a uniform rate. Whilst in method I. the observed value of **B** for a given value of **H** is that attained when the iron is first magnetised, the value observed by method II. is that attained after the iron has been carried through about six loops slowly, previous to which the current has been reversed about 100 times. The results appear to indicate that over this range the value of **B** attained for a given value of **H** increases as the magnetising force is carried through successive cycles, until finally a steady value of some 200 lines per square centimetre in excess of the initial value is attained.

Curves above **B** = 15,000. As considerable divergences were obtained in the upper regions of the magnetisation curves by the various methods examined, it was thought that the temperature of the iron might have some bearing on these discrepancies, for in some of the experiments the full magnetising current may have caused a rise of 100°C. in the worst case. To investigate this matter the magnetisation curve of the iron was taken with the specimen at normal temperature and repeated with the ring heated by means of an alternating current to a temperature above that which it was likely to have attained in any of the foregoing experiments (some 150°C.).

* To make this investigation more complete, it would be necessary to devote considerable time and attention to the experimental study of magnetic viscosity, and it was in the hope of being able to do this that the publication of this Paper has been delayed for nearly two years; leisure for this, however, has not been forthcoming, and consequently it is thought advisable to publish the investigation as it stands.

Results were obtained similar to those described by Dr. J. Hopkinson.* For small magnetising forces, say up to **H**=5, the permeability was increased, whilst for strong magnetising forces it was decreased, as a result of the rise of temperature. Portions of the magnetisation curves taken with the iron hot and cold are shown in Fig. 11. On the same diagram are plotted the points obtained as the maximum values reached in the "slow cyclic" hysteresis loops (method II.). It will be noticed that, as the limiting values of these loops increase, so

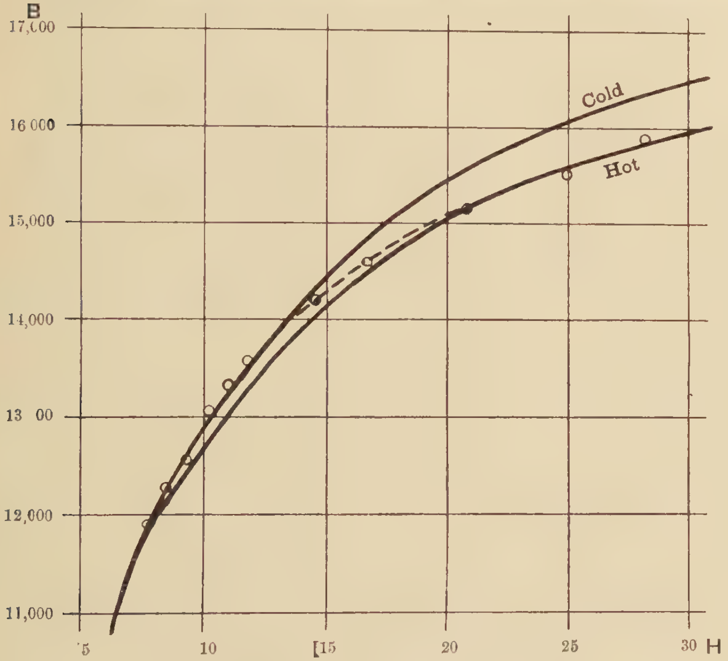

FIG. 11.—MAGNETISATION CURVES WITH IRON HOT AND COLD.

they gradually pass from the "cold" to the "hot" curve. The authors therefore conclude that the results obtained above **B**=15,000 are of little scientific value.

DEGREE OF ACCURACY.

In comparing the accuracy attainable by the various methods it is only necessary to consider the accuracy of the means employed to measure the flux-density **B**, the measurement of

* Hopkinson, "Original Papers," Vol. II., p. 186.

the magnetising force **H** resolving itself merely into a measurement of current, which may be made by the same means, whatever be the method used in determining the flux-density, except in the case of the alternating-current method.

Methods I. and II., Uniformly-varying Flux.—The measurement of flux-density by this method involves (1) the opposition of a constant P.D. to the E.M.F. induced in the secondary winding, and the measurement of this opposing P.D. ; (2) the maintenance of equality between the generated and opposing voltages ; and (3) the measurement of the time during which the flux is being changed.

With regard to (1), the opposing P.D. is obtained from a secondary cell connected to a high resistance, subdivided in a known ratio. Since the current given by the cell is very small, there is no difficulty in attaining constancy for the few minutes required for a test. The P.D. of the cell may be measured accurately by a potentiometer, and the opposing P.D. is then known with great accuracy from the known ratio in which the resistance is divided. As regards (2), the galvanometer used to ascertain the equality between the generated and opposing E.M.F.s may be as sensitive as desired. It is used merely as a " null " instrument, and its calibration curve is of no consequence. Since the secondary winding of the ring carries no current, it may be made of wire as thin as mechanical considerations will allow, and a large number of turns may be wound on. The rate of change of the flux may therefore be made small, without involving a very minute opposing E.M.F., and the change of flux represented by the smallest time measurable by the chronometric device used may be made as small as desired. In the case of (3), the accurate measurement of the time during which the flux changes presents no great difficulty, being of the order of from one to five minutes.

Methods III., IV. and V., Ballistic Methods.—The measurement of flux density by these methods depends on the observing of the throw given by a ballistic galvanometer, the calibration curve of which has to be determined with the aid of a mutual inductance. The accuracy attainable depends, therefore, on the accuracy of the mutual inductance, the constancy of the zero of the galvanometer, and also to a lesser extent upon the time period of the moving system of the galvanometer (8·6 seconds), the accuracy being greater as the time of swing is greater, owing to the movement of the swinging system being less in the time during which the impulse is being received.

Method VI., Alternating-current Method.—The accuracy of this method depends upon the accuracy with which the R.M.S. value of an alternating current can be measured, and also upon the accuracy with which the wave-forms of current and flux can be determined. By the use of a thermo-galvanometer the R.M.S. value of the magnetising current and of the induced E.M.F. may be measured with great accuracy, but the measurement of oscillograms presents difficulties owing to thickness of the line traced out on the plate by the vibrating beam of light and other causes, whereby the percentage accuracy in the measurement of the form-factors is not so great as that attainable in the measurement of currents.

Observation Errors.—In Tables V. and VI. are set out the differences observed between curves taken on different dates by the same methods in order to give an idea of the accuracy of repetition. In Table VII. are given the collected results expressed in percentage accuracy.

TABLE V.—*Accuracy of Repetition of Magnetisation Curve by method of Uniformly Varying Flux.*

Flux-density **B** by standard curve taken December 14 and 16, 1908	1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
Difference in **B** between this curve and curve repeated by same method February 9, 1909	+32	+32	+20	0	−10	+40	+62	+15	+80

TABLE VI.—*Accuracy of Repetition of Magnetisation Curve by Ballistic Methods.*

Flux-density **B** by method of uniformly varying flux (method I.)	2,000	4,000	6,000	8,000	10,000	12,000	14,000	16,000
Difference in **B** between curves taken May 5, 1908, and April 30, 1909, both by "step-by-step" method (method III.)	−40	−230	−140	−55	0	−50	+30	−160
Difference in **B** between curves taken August 4, 1908, and April 20, 1909, both by "method of reversals" (method V.)	+125	+80	−30	+25	+180	−20	−220	−29

TABLE VII.—*Comparison of Percentage Accuracy.*

—	1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
Uniformly varying flux	+3·2	+1·6	+0·7	0·0	−0·2	+0·7	+0·9	+0·2	+0·9
Step-by-step	...	−2·0	...	−5·7	...	−2·3	...	−0·7	...
Reversals	...	+6·2	...	+2·0	...	−0·5	...	+0·3	...

Effect of Intervening Temperature Rises on Accuracy of Repetition.—The authors have, after considerable experience, had forced upon them the fact that if determinations of permeability upon the same sample are to be repeated with an accuracy of even 5 per cent. on the lower portions of the magnetisation curves, it is essential that the ring be not subjected to considerable rises of temperature, say, of 100°C., even for short periods. Effects due to heating have been noted by many experimenters, such as S. R. Roget, in a Royal Society Paper.* Rises of temperature may be brought about by demagnetisation with alternating currents, or by the heat produced by excessive currents used with the object of determining the induction density at high values of **H**. The discrepancies observed at low values of **H** between two determinations made at the same temperature, but at different times, with an intervening period of heating, are not to be confused with the previously mentioned discrepancies in the higher regions of the magnetisation curves which are due to the differing temperature of the iron whilst the test is being conducted. If the iron be allowed to regain its original temperature after being heated, tests may be repeated accurately in the higher regions of the magnetisation curve.

Time Required for Tests by Different Methods.

As regards the time required for determinations by the various methods experimentally examined, ballistic methods are undoubtedly the most tedious, particularly when, for great accuracy, a ballistic galvanometer with a long period of swing is employed. The alternating-current method (method VI.) has considerable advantages in this respect, a full set of readings of magnetising current and induced voltage occupying a very short time. When, however, it is necessary to take oscillograms at various points in order to plot curves of form factors, the time required is enormously increased. The method of uniformly varying flux (method I.) is peculiarly adapted for use where time is a consideration, and at the same time a high degree of accuracy is desired. A complete magnetisation curve may be taken in a very few minutes, and the mean of many such curves obtained in, say, one hour, the iron being demagnetised between each test.

* S. R. Roget, "Effects of Prolonged Heating on the Magnetic Properties of Iron," The Electrician, Vol. XLI., 1898, p. 182.

Conclusion.

The method of "uniformly varying flux" appears to possess advantages, both scientific and practical, over the older methods in use for the testing of ring samples of magnetic materials. It avoids difficulties due to eddy currents and magnetic viscosity, which effects are themselves due primarily to rapid or irregular changes of flux. It also has the practical advantage that experiments may be carried out with rapidity, accuracy of repetition, and under standard or predetermined conditions of magnetic change.

The authors, therefore, would commend this method to the careful consideration of those interested in the carrying out of magnetic tests, especially where great accuracy under definitely known conditions of experiment are essential.

ABSTRACT.

The research described was instituted with the object of finding what differences there were between the magnetisation curves for a given sample of iron when determined (1) by the older methods in which the flux is changed suddenly, (2) by a method in which it is changed exceedingly slowly and at a uniform rate.

The methods experimentally examined were :—

1. Method of constant rate of change of flux.
2. "Slow cyclic" hysteresis loops by method 1.
3. "Step by step" magnetisation curve.
4. "Step by step" hysteresis loops.
5. Method of reversals.
6. Alternating-current magnetisation curve.

Details are given of the theory and practical working of method 1. In this method the magnetising current is continuously increased through a primary winding by a specially designed resistance at such a varying rate as to maintain a constant voltage generated in a secondary winding. A certain amount of skill is required in operating the resistance, but an average experimenter may easily acquire this with a little practice. The complete change of the current occupied times varying from one up to some five minutes.

Tables are given of the magnetisation curve determined by the six different methods at a number of values from H 0·3 up to 70·0, and of the permeability from B 500 up to 17,000.

At low values of the magnetising force the uniformly varying flux method gives results of some 200 lines per square centimetre in excess of the older methods.

As regards the time required for determinations by the various methods experimentally examined, ballistic methods are undoubtedly the most tedious. The alternating-current method (method 6) has considerable advantages in this respect, a full set of readings of magnetising current and induced voltage occupying a very short time. When, however, it is necessary to take oscillograms at various points in order to plot curves of form factors, the time required is enor-

mously increased. The method of uniformly varying flux (method 1) is peculiarly adapted for use where time is a consideration, and at the same time a high degree of accuracy is desired. A complete magnetisation curve may be taken in a very few minutes, and the mean of many such curves obtained in, say, one hour, the iron being demagnetised between each test.

The method of "uniformly varying flux" appears to possess advantages, both scientific and practical, over the older methods in use for the testing of ring samples of magnetic materials. It avoids difficulties due to eddy currents and magnetic viscosity, which effects are themselves due primarily to rapid or irregular changes of flux. Besides rapidity of experiment, it also has the advantage of accuracy of repetition under standard or predetermined conditions of magnetic change.

The method is therefore commended for the carrying out of magnetic tests, especially where great accuracy under definitely known conditions of experiment are essential.

DISCUSSION.

Mr. A. Campbell remarked that it was rather difficult to say whether it was tests made with a slowly varying flux that were required in practice. He thought tests made with cyclical changes were more useful. The differences in the results given by the two methods were important. He pointed out that the method could be used for the determination of the ohm by substituting a known mutual induction for the iron under test.

Dr. A. Russell pointed out that the value of **H** was different at the inner and outer radii of the iron ring used by the authors.

Dr. S. W. J. Smith said it would have been interesting and instructive if the authors had performed a similar series of experiments by the different methods upon some material freer from magnetic viscosity than soft iron. Experiments made upon a sample of nearly pure iron in weak fields did not seem to be explicable along the lines suggested by the authors for the iron they used.

Prof. C. H. Lees drew attention to the extraordinary simplicity of the reluctivity curves. It looked as if $1/\mu$ were a linear function of **H**.

The President remarked that by superposing on the constant magnetising field an alternating field which was gradually reduced to nothing, much larger values of the apparent permeability were obtained. For weak fields it was over 100,000. This effect was due to the residual magnetism left by the alternating field.

Prof. J. T. Morris, in reply, stated that they had not tried their method at widely different speeds. In reply to Dr. Russell, he stated that the same iron ring was tested by the different methods. Replying to Mr. Campbell, he had thought of the present method as a means of measuring a mutual induction.

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